Shape Spaces
Les Espaces de Formes

Master Course - February, 4th, 2014 - Paris - France

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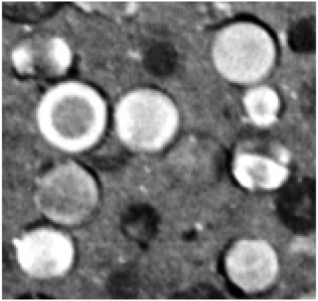
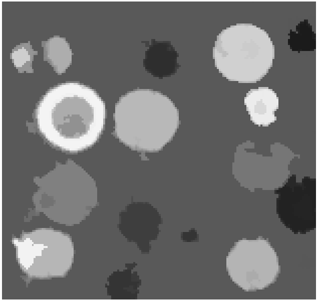
Motivation



Input image.



Result.

Question

How to obtain such a result?

Outline

Outline

Outline

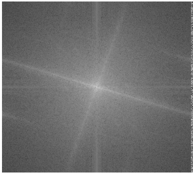
Outline

Outline

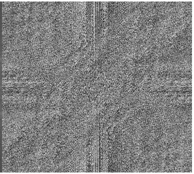
Image representations

Decomposition into primitive or fundamental elements that can be more easily interpreted:

- Functional decompositions;
- Multiresolution decompositions;
- Multi-scale representations;
- Threshold decompositions;
- Hierarchical representations.



Amplitude



Phase

Image representations

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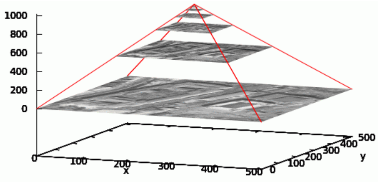

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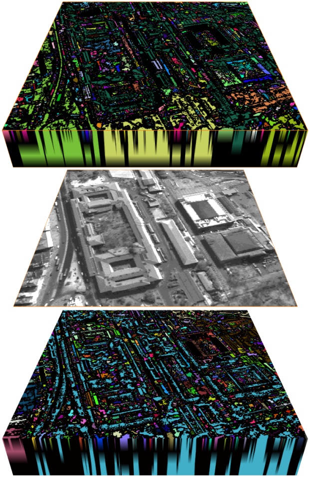

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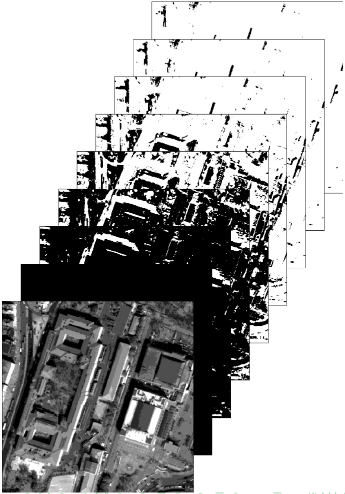

Image representations

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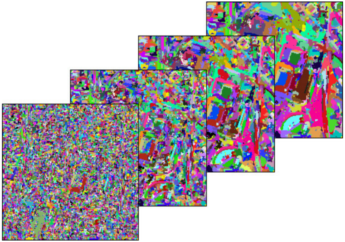

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Image representations

Decomposition into primitive or fundamental elements that can be more easily interpreted:

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- Hierarchical representations.

Not mutually exclusive.
Properties inherited from those of underlying operations.
Choice driven by the application needs.

Connected operators

What's connected operators ?

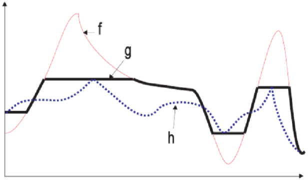
Filtering tools that merge flat zones.

Properties

- No new contours,
- Keep contours' position.

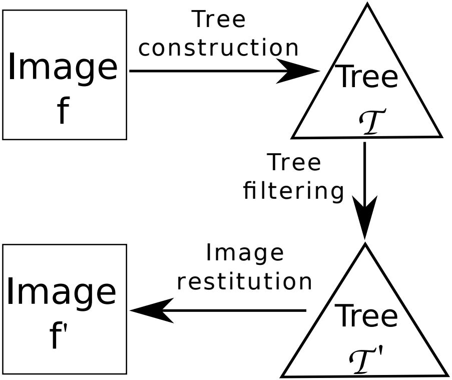
An example : Levelings

Lower-leveling: for x and y neighbors, $g(x) > g(y) \Rightarrow g(y) \geq f(y)$.
Upper-leveling: for x and y neighbors, $g(x) > g(y) \Rightarrow g(x) \leq f(x)$.
Leveling: Lower-leveling $\cap$ Upper-leveling.

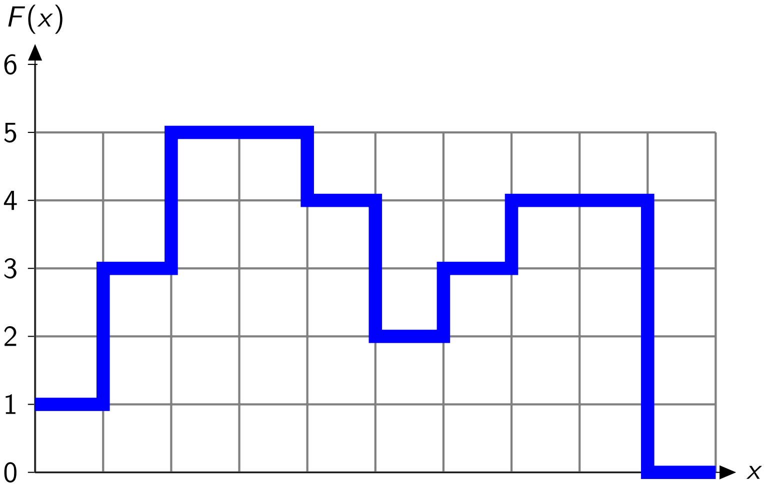


Leveling with marker.
f : input,
h : marker,
g : result.

One popular implementation [Salembier & Wilkinson, SPM, 2009]

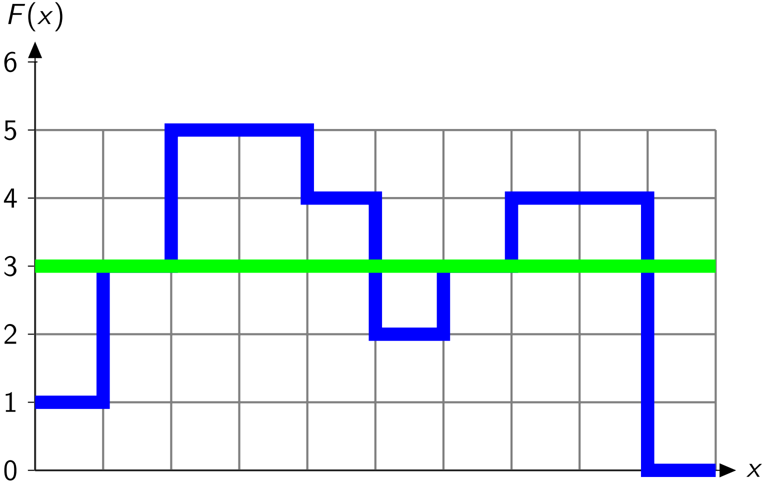


Level sets and components



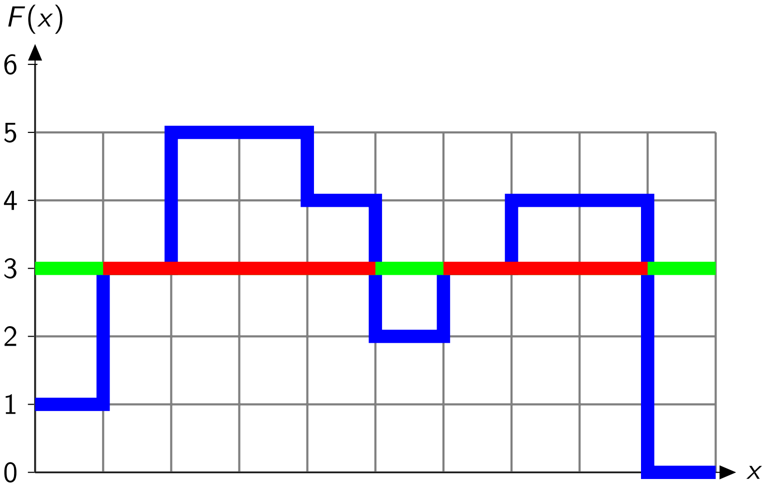
$$y = F(x).$$

Level sets and components



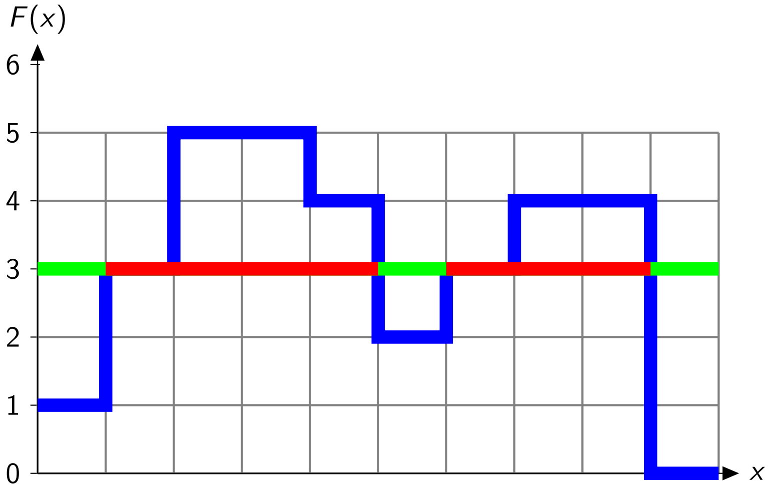
$F(x) \geq 3.$

Level sets and components



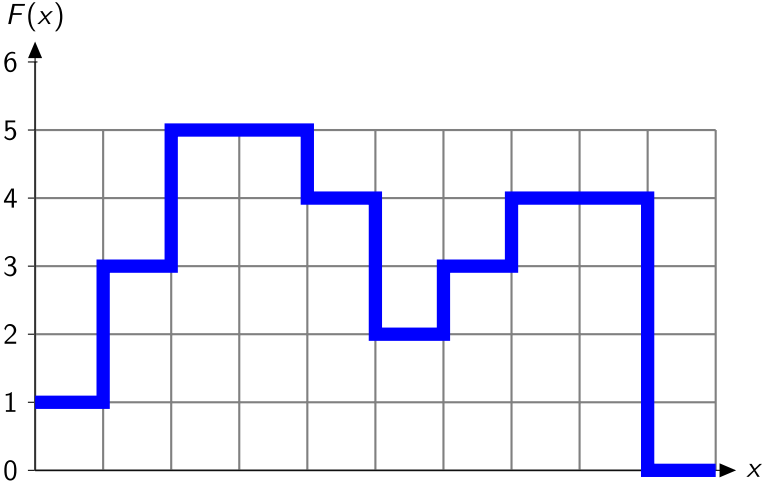
$$F_3 = \{x \mid F(x) \geq 3\}.$$

Level sets and components

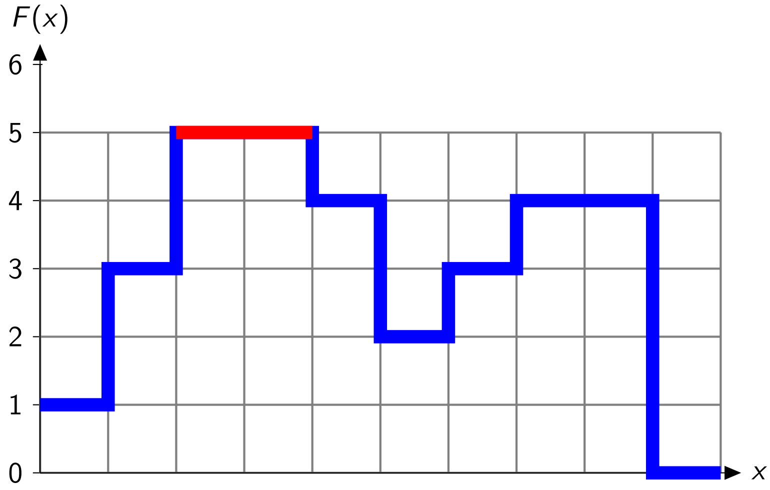


$$F_k = \{x \mid F(x) \geq k\}.$$

(Max) component tree

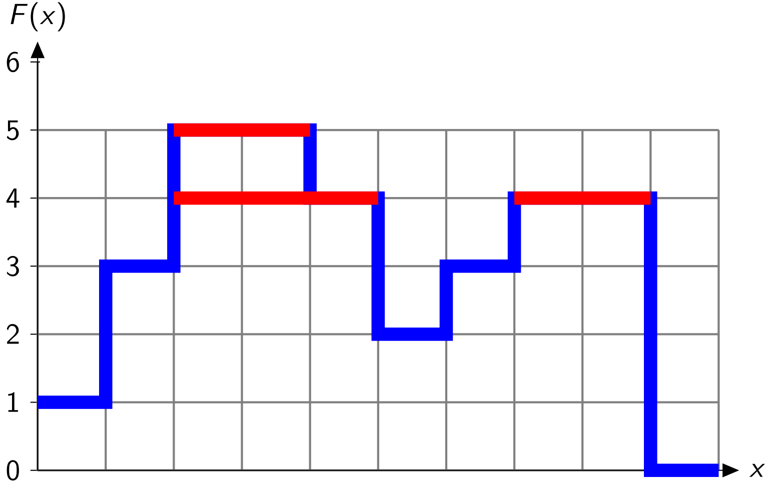


(Max) component tree



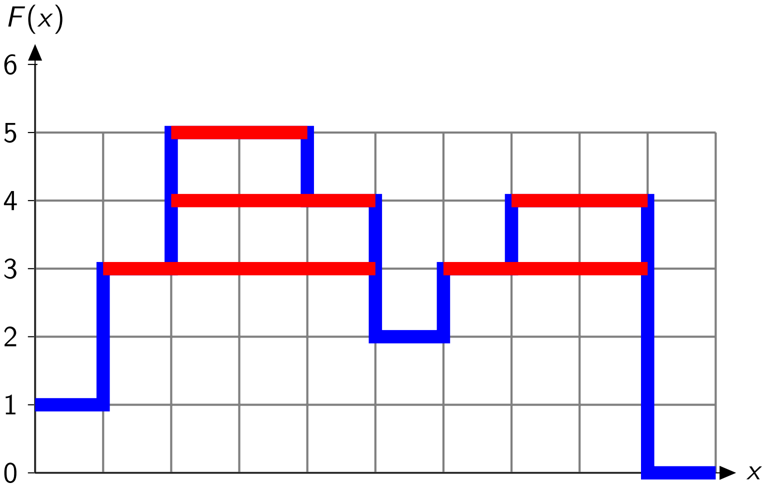
$F_5 = \{x \mid F(x) \geq 5\}.$

(Max) component tree



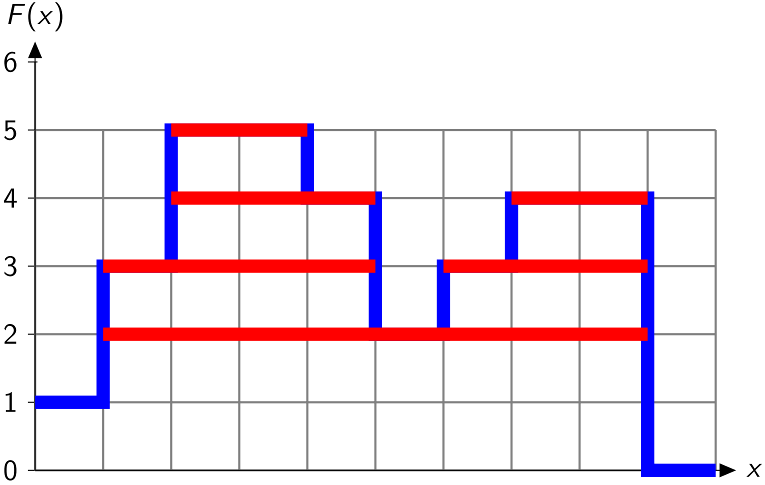
$F_4 = \{x \mid F(x) \geq 4\}$.

(Max) component tree



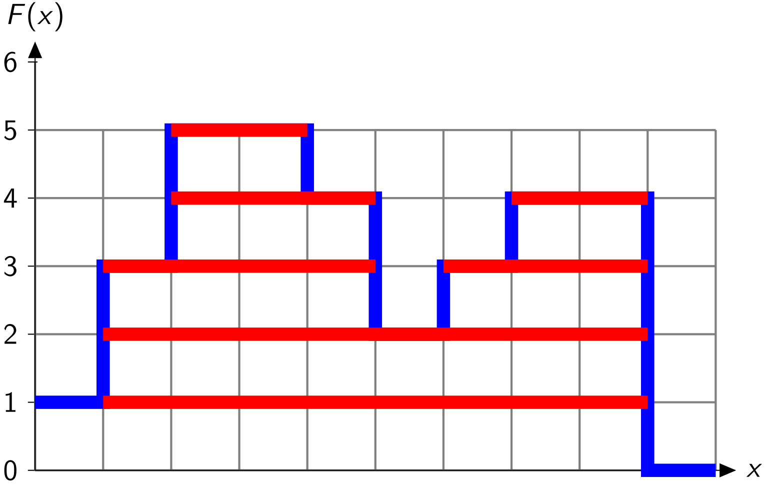
$$F_3 = \{x \mid F(x) \geq 3\}.$$

(Max) component tree



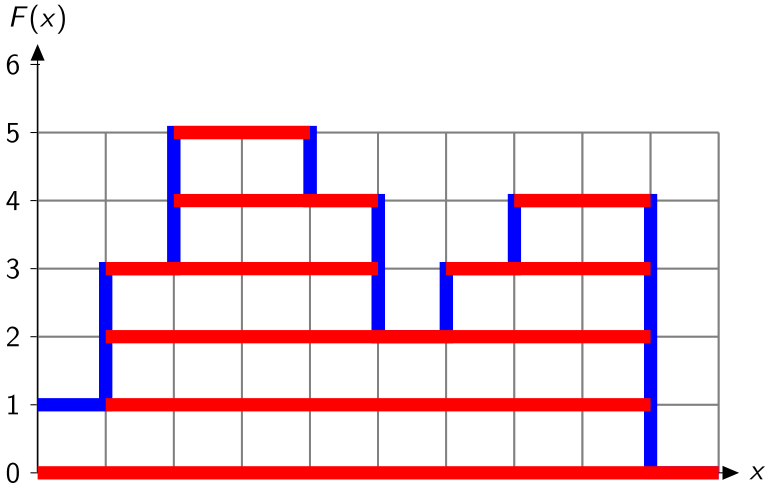
$F_2 = \{x \mid F(x) \geq 2\}$.

(Max) component tree



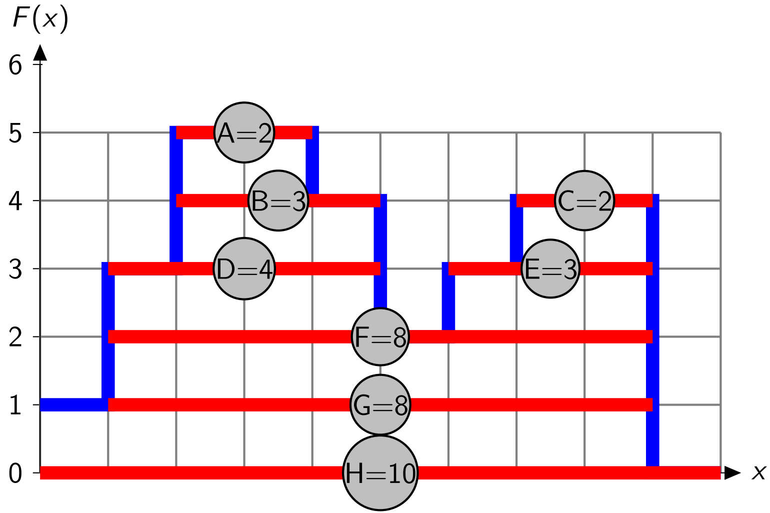
$$F_1 = \{x \mid F(x) \geq 1\}.$$

(Max) component tree

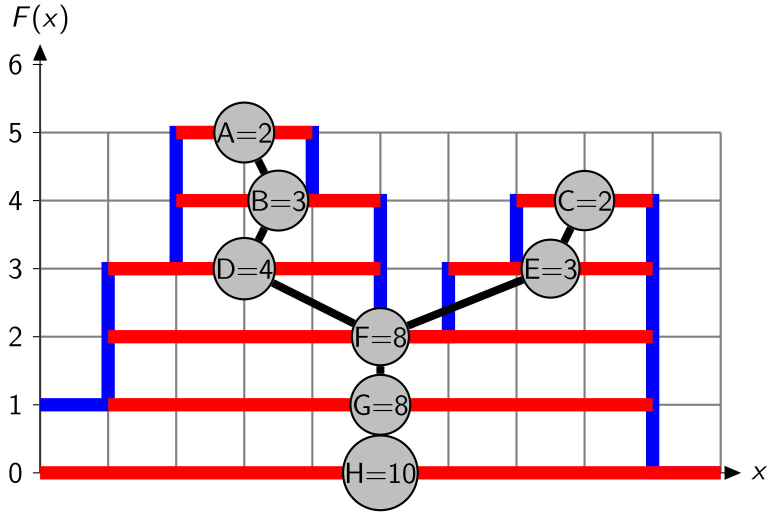


$F_0 = \{x \mid F(x) \geq 0\}$.

(Max) component tree

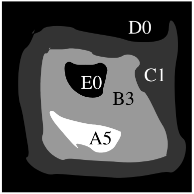


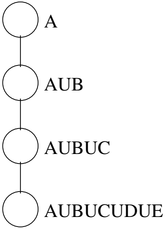
(Max) component tree



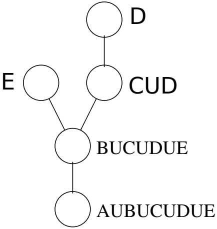
Components + inclusion relationship = component tree.

Some of the many possible trees

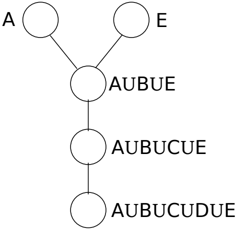




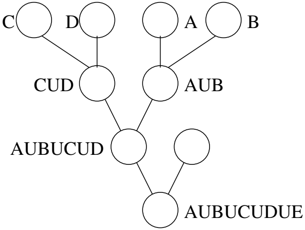
Max–tree



Min–tree



Tree of shapes



Binary Partition Tree

Some link with Morse's Theory

Important idea

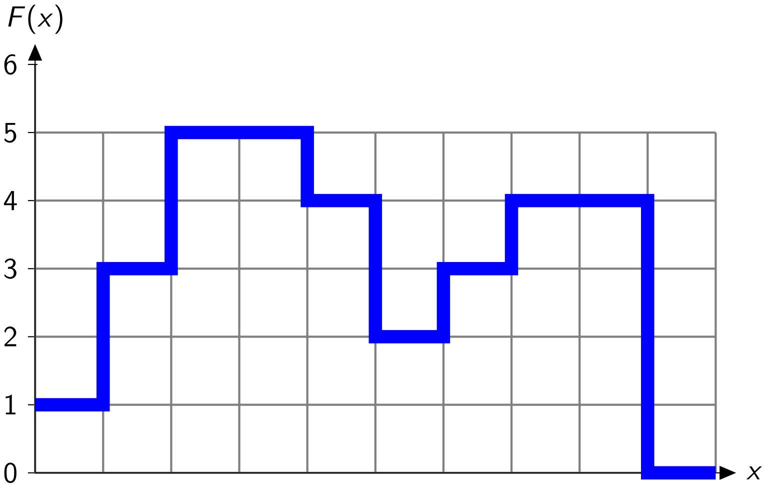
Some nodes are more important than others

- *Leaves* $\Leftrightarrow$ *"Extrema"*
- *Nodes with more than one child* $\Leftrightarrow$ *"Saddle points"*
- *Hence, filtering is linked with* topological persistence

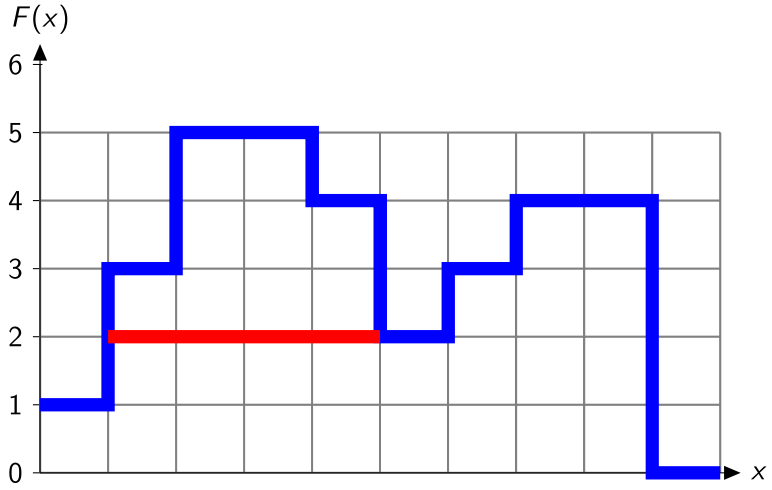
Shape spaces: what are they?

- A family of shapes (first part of the talk)
  - One can process each shape individually (keep/remove/highlight)
  - with any criterion, attribute, energy . . . => **NOT ROBUST**
- With a tree structure (first part of the talk)
  - A first "topology" on the family of shapes
  - Increasing criterion
- With a graph structure (second part of the talk)
  - A more complete structure on the family of shapes
  - Generalization of the previous approaches
  - With any criterion, attribute, energy . . . => **ROBUST**

Attributes

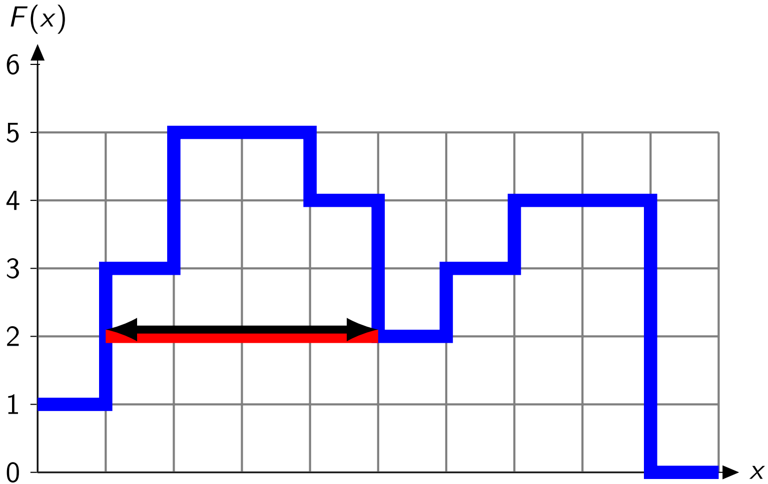


Attributes



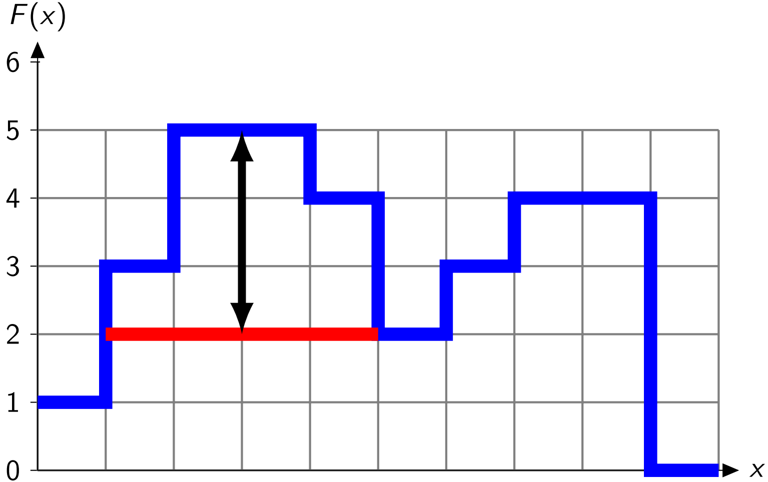
A connected component.

Attributes



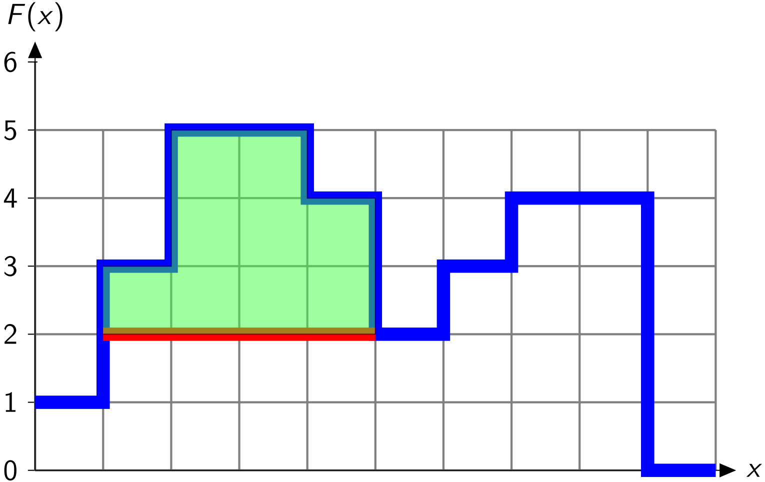
Area.

Attributes



Height.

Attributes



Volume.

Attributes

Increasing attributes

Increasing attributes : $A \subseteq B \Rightarrow \mathcal{A}(A) \leq \mathcal{A}(B)$.
Examples : Area, height, volume.

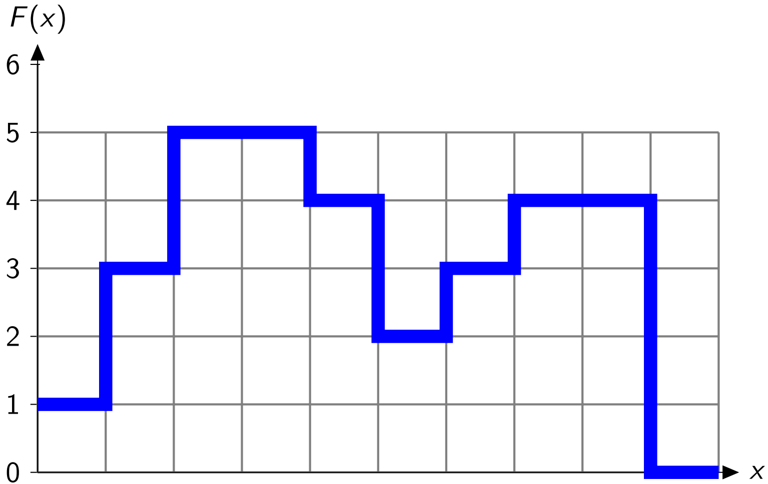
Non-increasing attributes

Shape attributes.

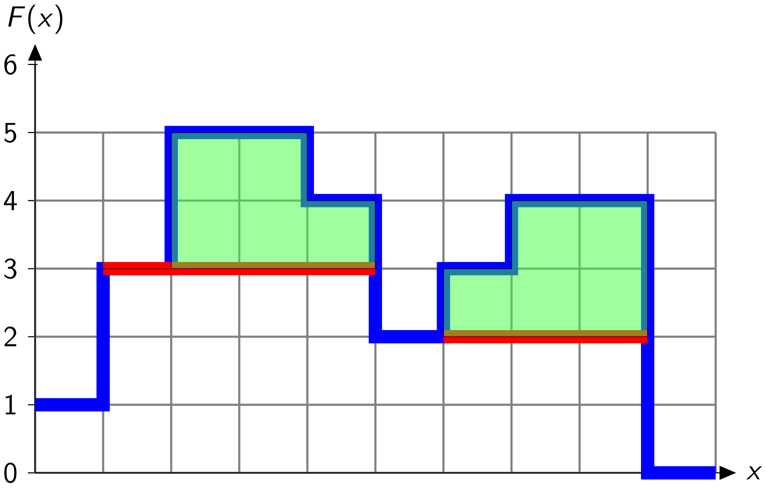
- I/A^2 minimum for a round object,
- Circularity : $area/(\pi \times l_{max}^2)$,
- Elongation : L_{max}/L_{min}.

L_{min} and L_{max} : Length of the two main axes of the best fitting ellipse.

Filtering with increasing attributes

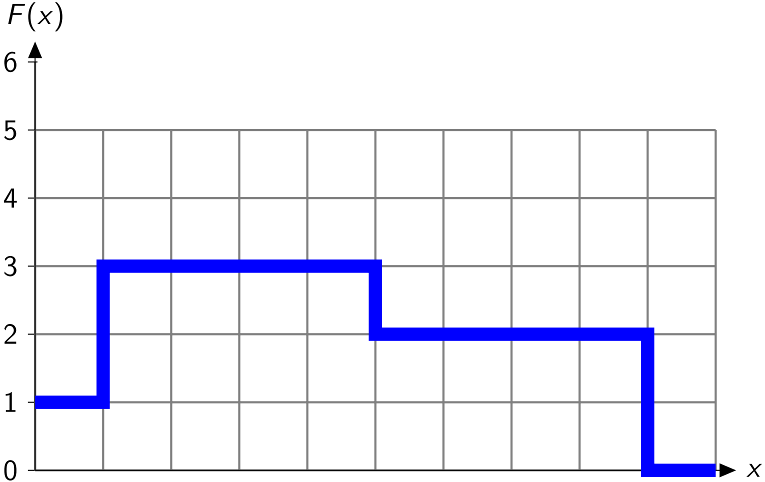


Filtering with increasing attributes



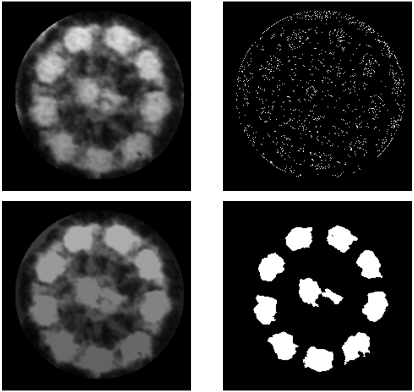
Volume ≤ 5.

Filtering with increasing attributes



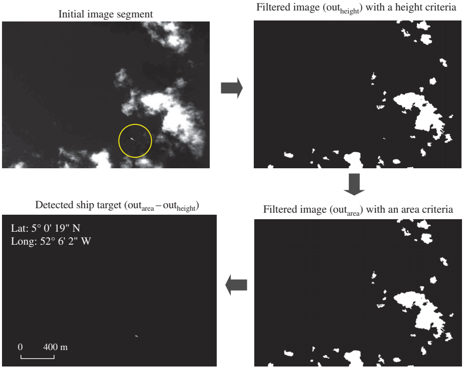
Filtered function.

Applications: filtering with increasing attribute



The 9+2 microtubule doublets of a motile cilia
Attribute: Volume

Applications: filtering with increasing attribute



Ship detection on optical satellite image

C. Corbane *et al.*, International Journal of Remote Sensing 31 (22), 5837-5854

Applications: filtering with increasing attribute

FIRST

future. According to Dealogic, 87 SPACs have begun trading in the U.S. since the end of 2003, buying some well-known companies like smoothie purveyor Jamba Juice. Last year alone, 40 SPACs worth $3.4 billion were announced, up from $484 million two years earlier.

SPACs are essentially shell companies. They go public with little more to show investors than a management team and an agreement that the money raised will be used to fund an acquisition in a particular sector, such as retail, or in an emerging market like China. Ergo, a SPAC is a roll of the dice. "Imagine paying $50 to go to a Broadway show, and you have no idea what's behind the curtain," says lawyer Mitchell Littman of Littman Krooks LLP, who works with a lot of SPACs. "You are relying on the fortune and integrity of the management team." Once a deal is completed, the SPAC's managers (who typically receive 20% of the public shares as compensation) are free to sell their holdings, usually after a lockup period.

Today's SPAC boom harkens back to the flurry of "blank check" companies that sprang up in the 1980s. Those were widely discredited by a wave of scams in which fraudsters would take a shell company public, announce a merger, pump the stock, and then dump it before everyone realized that the hot target company was anything but. Investors have a lot more protection with SPACs, which must hold almost all the money in escrow until a deal is done.

From Charney's perspective, there is no downside to the arrangement. It lets him raise cash relatively quickly, remain the largest shareholder, and avoid having to sell other investors on his vision until after his company is already publicly traded. "I'm in a hurry to get the money," he says, citing his plan to expand beyond his current 145 stores. "What's great about the SPAC is, it's kind of in reverse. First you get publicly traded, then you do the compliance stuff. And if you brought in private equity, they might want to run the company in a more [cautious] manner," he says, referring to American Apparel's risqué image.

FIRST YOU GET PUBLICLY TRADED, THEN YOU DO COMPLIANCE.

Keeping control is particularly important for a guy who is the creative force behind the brand, as well as one whose flamboyant management style—there is one sexual-harassment suit outstanding against him by a former employee; two others have been dismissed—might scare off many investors. "He is very talented," says former J.C. Penney CEO Allen Questrom, a partner in private-equity firm Lee Equity Partners, "but he is a *meshugana* [read: nutty] type guy. It's very difficult to see him in a public sphere." (Charney, for his part, dismisses the talk about his reputation as "all tabloid.")

Charney's new bedfellows don't appear concerned, even if they're more likely to wear dark suits than American Apparel's colorful leggings. Endeavor's president, Jonathan Ledecky, is familiar with unusual financing vehicles; he founded U.S. Office Products, the once-hot roll-up that went bankrupt in 2001 (long after Ledecky cashed out). Boldfaced names on the board include Kerry Kennedy, Bobby's daughter, and Edward Mathias, a managing director at the Carlyle Group. On Jan. 31 the same group formed a new $250 million SPAC, Victory Acquisition, underwritten by Citi. The filing notes that it could compete directly with Endeavor. And as with Endeavor, once Victory goes public it will pay a Ledecky "affiliate" $7,500 monthly to run its Manhattan office.

Hedge funds are drawn to these investments. Since SPACs typically must announce a deal within 18 months of their IPOs or return the money raised, with interest, hedgies see them as safe places to park cash while potentially profiting from the price fluctuations between a deal's announcement and its close—a sort of arbitrage play. That may be why Steven Cohen's $12 billion hedge fund SAC Capital snapped up 8.7% of Endeavor's stock when the deal was announced.

Even if investors are SPAC-happy, not all regulators are: So far only the American Stock Exchange lists SPACs, and many bankers and lawyers say the SEC is deliberately taking its time vetting deals, hoping to slow down the pace of offerings. SPACs are probably best left to those, like Charney, who like to live life on the edge.

SPAC Attack!

The volume of special purpose acquisition companies going public in the U.S. is rising sharply.

Number of deals
Value of deals (in billions)
13 $0.5 2004
29 $2.1 '05
40 $3.4 '06

UNHEALTHY RETURN Since smoothie chain Jamba Juice went public in November via a SPAC, its shares have fallen 19%.

FORTUNE CHART / SOURCE: DEALOGIC; PHOTO: DAVID MCNEW—GETTY

12 • FORTUNE February 19, 2007

Box detection on a document image

Attribute: (width, height) of the component

Applications: filtering with increasing attribute

AFRICA

Emotional Intelligence

Soul City, South Africa's wildly popular soap opera, spreads public-health messages across the continent

By PETER HAWTHORNE JOHANNESBURG

IT'S 8:30 ON A TUESDAY NIGHT, AND MILLIONS of South Africans are tuning in to find out if the white social worker, Karen, will be able to adopt AIDS orphan Benni—"People who love me always end up dying," the child says. No, it's not *Melrose Place* or *E.R.* or any of the other imported dramatic series that fill the country's airwaves. It's a show about South Africans themselves. *Soul City*, the local soap opera that began as a good deed, is now an award-winning, multi-media business internationally recognized for its role in Third World development. It is also one of the top three most widely watched progams on South African TV.

Soul City reflects the life and hard times of people in a typical South African urban black township. In fact, most of the series' location shots are taken in Alexandra, one of the oldest and most squalid of South Africa's black slums, just outside Johannesburg. And *Soul City's* clinic, where much of the drama is centered, is based on an actual clinic in Alexandra.

That medical theme is not surprising, since the idea for the show came from two young white South African doctors, Garth Japhet and Shereen Usdin, who had studied and worked in township hospitals and clinics, including in Alexandra. They envisioned a dramatic program that would highlight the social and health problems—particularly HIV-AIDS—in the townships. They convinced a long list of donors—which today include the European Union, Britain's Department for International Development, UNICEF, British Petroleum, the mobile-phone network MTN and the South African government's Department of Health—to put up the money, and *Soul City* was launched nationwide on the South African Broadcasting Corporation's TV service in 1994. "We realized that to be effective we had to go out at prime time, we had to keep it going and we had to be as good as or even better than any other production," says Japhet. Since then, *Soul City* has collected a bundle of prizes. On June 14, the show received a special award from Britain's One World Broadcasting Trust for its role in "illuminating the concerns of developing societies."

Because AIDS is a huge problem in South Africa—almost 10% of the population is infected with HIV—the subject is featured in every installment of the show. Other topics include violence against women, drunkenness, rape, child abuse, sexual conduct, occupational health, disability, setting up a business, even buying and selling a used car. "This is not just about reality in life, it's about real-life skills," says Japhet, who is executive director of the nongovernmental Institute for Health and Development Communications, which produces the show. *Soul City* also has a radio version broadcast daily in nine of South Africa's 11 official languages, as well as weekly newspaper columns and a series of educational booklets.

Characters speak in several of South Africa's official languages, and the series is subtitled in English. It has been shown in Zimbabwe, Zambia, Nigeria, Namibia, Malawi and Barbados and has been dubbed into French and Portuguese for showing in Ivory Coast, Mozambique and other countries. "This concept has got legs," says Japhet. "It moves."

It has also spun off *Soul Buddyz*, a show for younger viewers, which began a 26-episode run on SABC-TV last year. Much as *Soul City* does for adults, *Soul Buddyz* looks at AIDS and other problems as they affect the young. The show is already being sought by countries outside South Africa.

What makes *Soul City* so successful is not merely its authentic portrayal of township life but also its intimacy with viewers. Many of the actors live in the townships. "That sometimes presents a problem," says Lebo Ramafoko, producer of the latest series. "Our actors are seen as real people. They have to become the advocates and counselors they portray on the screen." Some of the cast members have been discovered through *Soul City's* annual "Search for a Star" initiative. Among them is Doreen Kotsedi-Pitse, a blind woman who will play the part of a radio talk-show coordinator. Kotsedi-Pitse lost her sight as a result of domestic violence, an issue frequently raised by *Soul City*. "It made us all aware how uncomfortably close we are to real life," says Ramafoko.

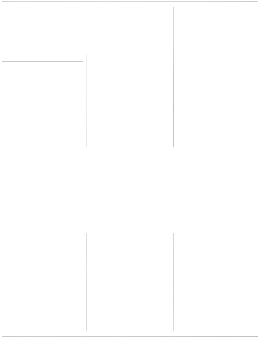
Soul City is affecting the way South Africans lead their real lives. A study sponsored by the E.U. says there is evidence that the program has played "a major role" in increasing public knowledge of HIV-AIDS and safer sexual behavior. Researchers say, for instance, that there is a "significant" association of the use of condoms with people who watch *Soul City*. The survey did not say whether Benni would find a happy new home. For more about that, tune in to next week's episode. ■

IT'S A WRAP: Actors on the set take a breather from filming after a successful take

HIGH DRAMA: Confronting prejudices toward HIV is one of Soul City's main objectives

EDUCATIONAL FOCUS: Benni tries his hand learning sign language at the township clinic

38 TIME, JULY 2, 2001



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Operator: top-hat

Applications: filtering with increasing attribute

Briefing

Verbatim

'Nobody is going to be allowed to do anything here.'
WAHEED ARSHAD, Pakistani Major General, on reports that the U.S. may expand military and CIA authority to pursue Islamic militants in Pakistan

'If you are a believer in miracles, this would be one.'
PHILIP S. BARIE, chief of the division of critical care at New York–Presbyterian Hospital, on the recuperation of Alcides Moreno, a 37-year-old window washer who fell 47 stories last month

'We should not resort to violence even if we have differences.'
MAUMOON GAYOOM, President of the Maldives, after an attempt to assassinate him was foiled when a boy scout waiting to greet the island nation's leader grabbed the attacker's knife as he lunged out of a crowd

'This is who I am. This is my life.'
SERGEANT DARREN MANZELLA, U.S. Army medic and a decorated veteran of Iraq, discussing his homosexuality in a Jan. 8 press conference. Manzella has been openly gay since August 2006, despite the military's "Don't ask, don't tell" policy; its critics have argued that his case proves the policy is arbitrarily enforced

'They should either kill me or organize a second round of elections.'
LEVAN GACHECHILADZE, Georgian opposition leader, after his resounding defeat by pro-Western incumbent Mikhail Saakashvili in the country's Jan. 5 Presidential elections. Gachechiladze claimed the voting was rigged

'This is the first red carpet that I've really walked down where I didn't have to think about holding in my stomach.'
HALLE BERRY, on being pregnant, while accepting the Desert Palm Achievement Award at the Palm Springs International Film Festival on Jan. 5

MORE AT TIME.COM
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Sources: A.P.; New York Times; A.P. (2); BBC; PEOPLE

NUMBERS

CENSORSHIP
16,600%
Percentage by which Burmese authorities have increased the license fee to receive satellite TV, the country's only broadcast-news source not controlled by the state

300%
Percentage by which the new fee—about $800—surpasses the annual salary of a Burmese schoolteacher. The increase is seen as a move to block access to news critical of the ruling military junta

HEALTH
65%
Percentage of U.S. adults who are exercising, dieting or both during any nonholiday period

29
Number of pounds (13 kg) that at least 25% of American adults currently on a diet would like to lose. Regular exercisers said they want to drop an average of 14 lbs. (6.3 kg)

GOLD
$884
Record price per ounce of gold set on Jan. 8, as investors reacted to market turbulence and high oil prices, beating the previous all-time record of $875 set in 1980

$2,200
1980's record price for gold, when adjusted for inflation

WEALTH
$46,380
The U.K.'s per capita gross domestic product this year, $500 more than U.S. per capita GDP. For the first time since the 19th century, the average Briton will earn more than the average American

1993
The year following Britain's last major recession, when the country's per capita GDP was 34% less than that of the U.S.

Sources: Reuters; A.P.; Pew Research Center (2); International Herald Tribune; A.P.; BBC (2)

12

TIME January 21, 2008

Letter detection on a document image

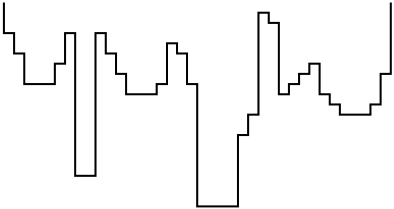
Attribute: area of the component

Operator: top-hat

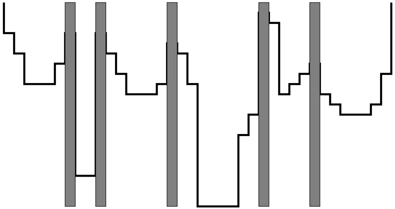
Some functions and spaces on which to compute the trees

- An image
- A gradient
  - Especially usefull with the watershed
- A node-weighted graph
- An edge-weighted graph
- A weighted mesh
- A density function
  - Topological mean-shift filtering
- and more in the sequel

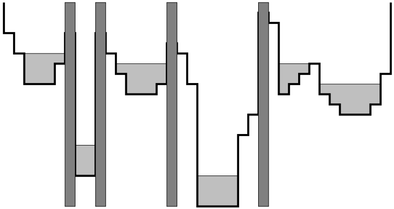
Hierarchies: floodings and watersheds



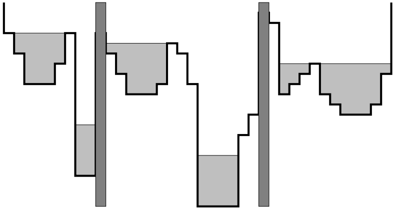
Hierarchies: floodings and watersheds



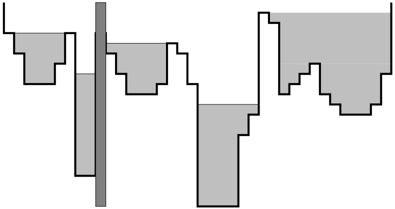
Hierarchies: floodings and watersheds



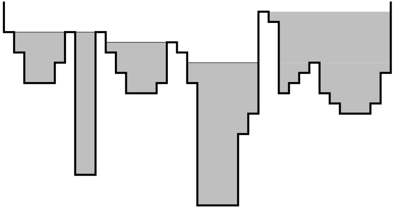
Hierarchies: floodings and watersheds



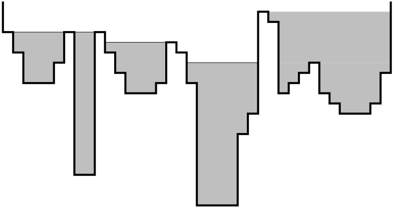
Hierarchies: floodings and watersheds



Hierarchies: floodings and watersheds



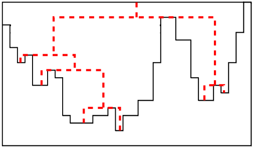
Hierarchies: floodings and watersheds



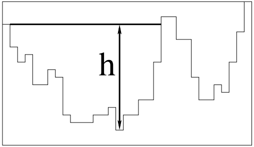
Important idea

- *There exists numerous criterions for flooding a surface.*
- *Flooding can be done through the min-(component-)tree.*
- *Among those criterions, notably: depth, surface, volume.*
- *[Beucher, ISMM, 1994 - Najman & Schmitt, PAMI, 1996 - Meyer et al., An. Telecom, 1997]*

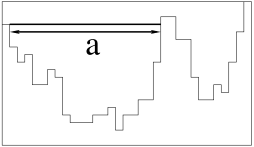
Flooding and the min-tree



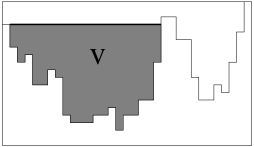
(a) Component tree



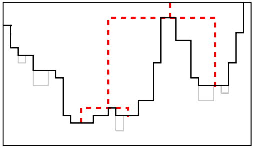
(b) Height



(c) Area

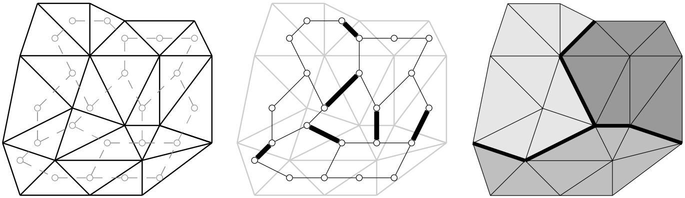


(d) Volume

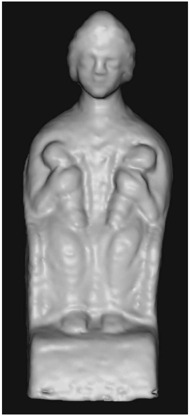


(e) Area filtering

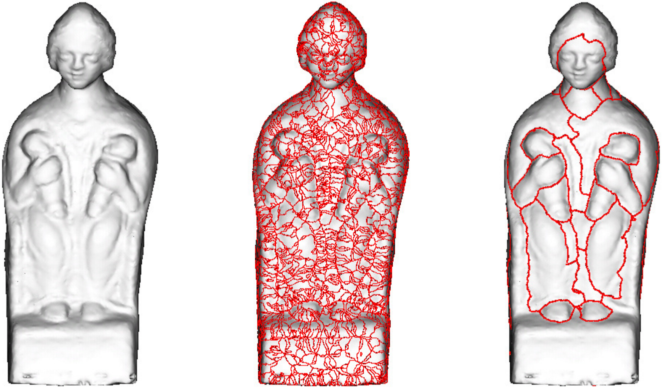
An example of mesh segmentation



An example of mesh segmentation

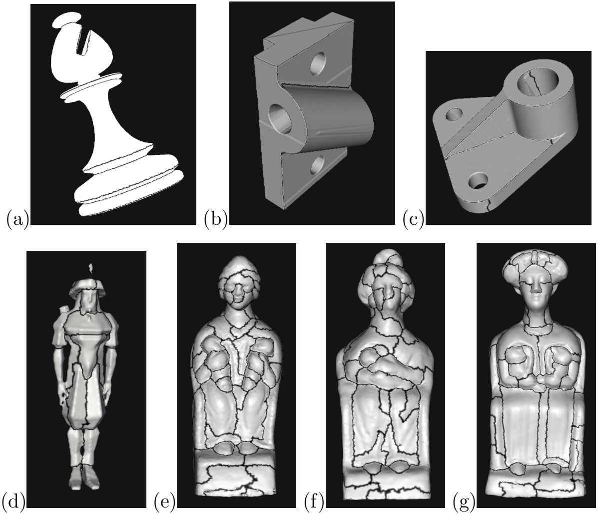
(a)  (b)  (c) 

An example of mesh segmentation



S. Philipp-Foliguet *et al.*, Pat. Rec., 2011, 44 (3), pp. 588-597

An example of mesh segmentation



A topological mean-shift algorithm [Paris-Durand CVPR 2007]

Filtering with increasing attributes

Pruning the trees

$\mathcal{A} \uparrow$, Pruning the leaves = Attribute thresholding.

Non-increasing attributes

How to process the filtering?

Filtering with non-increasing attributes [Salembier & Wilkinson, SPM, 2009]

Pruning strategies

- *Min*,
- *Max*,
- *Viterbi*.

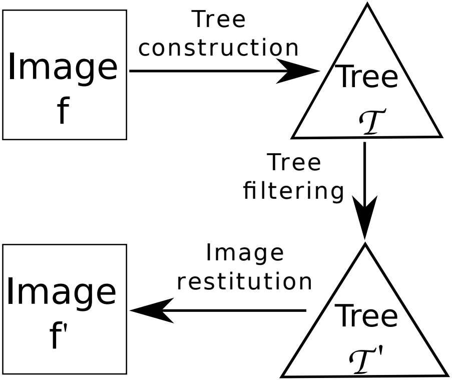
Remove the sub-tree rooted in the node.

Attribute thresholding strategies

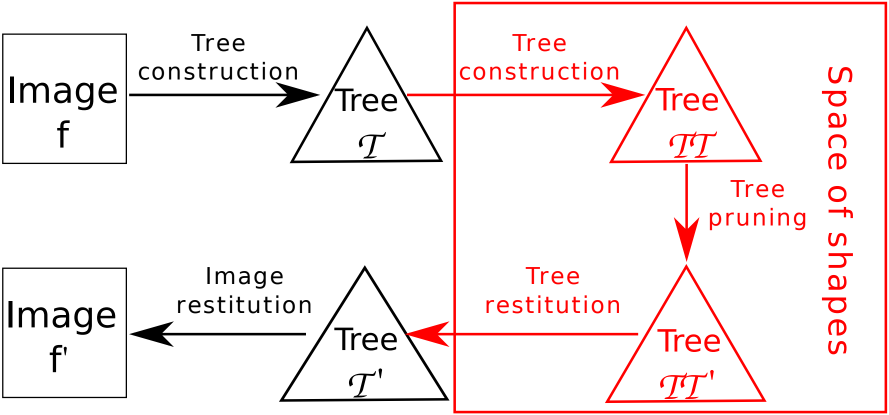
- *Direct*,
- *Subtractive*.

Remove the nodes under the threshold.

Our proposed framework

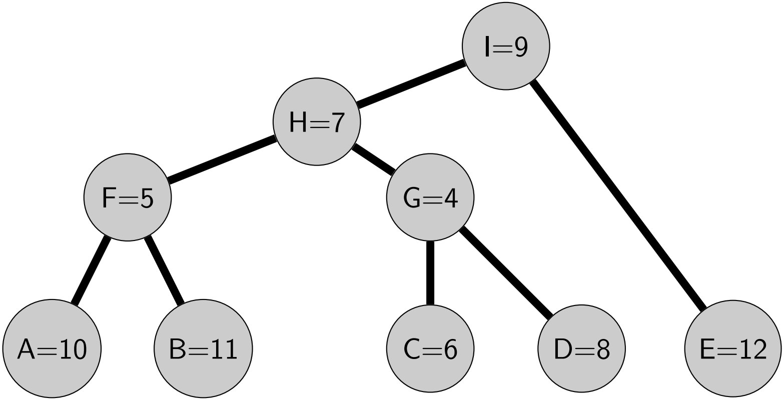


Our proposed framework [Xu & Géraud & Najman, ICPR, 2012]

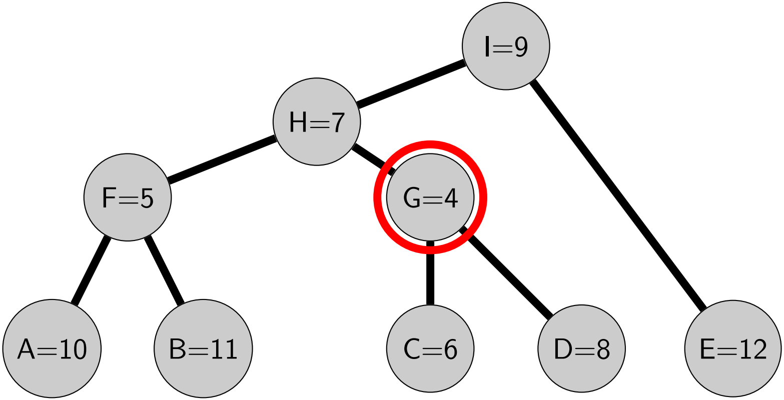


Outline

Construction of second tree representation

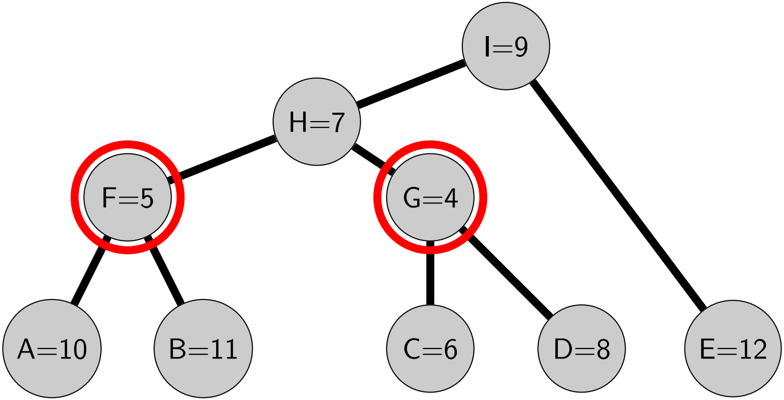


Construction of second tree representation



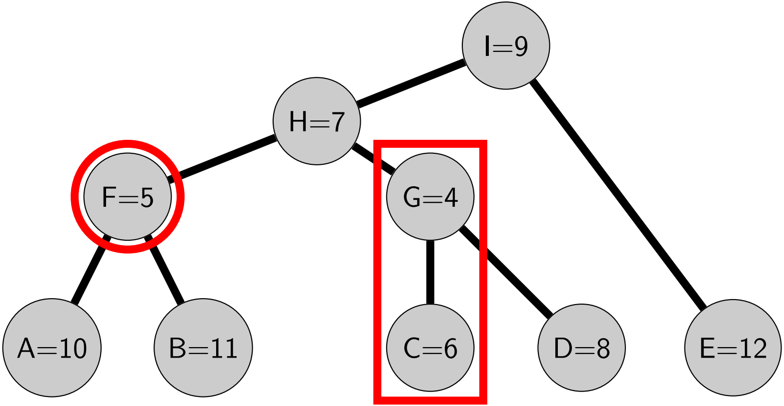
Level $\{x|A(x) \leq 4\}$.

Construction of second tree representation



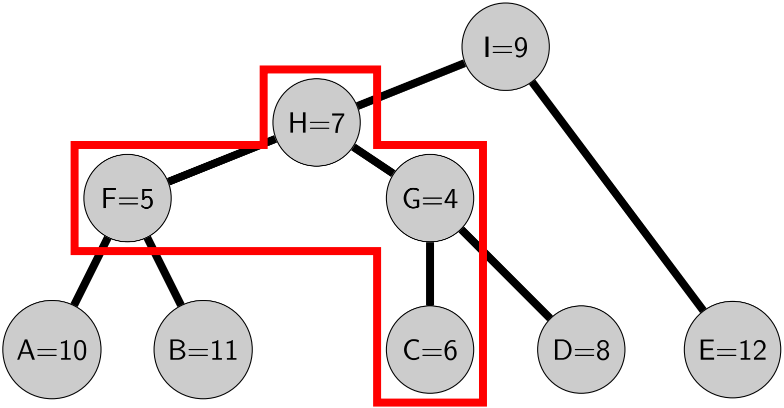
Level $\{x|A(x) \leq 5\}$.

Construction of second tree representation



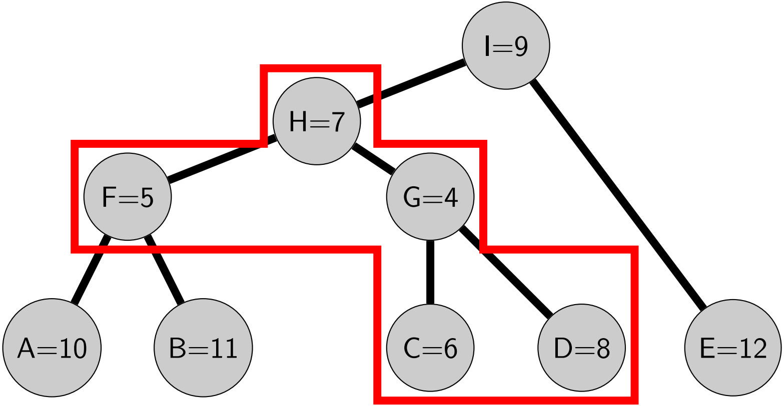
Level $\{x|A(x) \leq 6\}$.

Construction of second tree representation



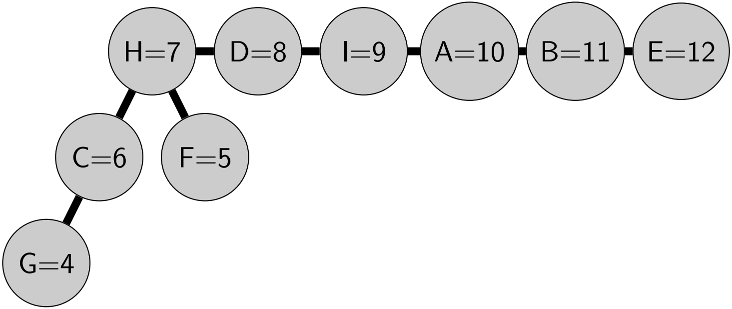
Level $\{x | A(x) \leq 7\}$.

Construction of second tree representation



Level $\{x|A(x) \leq 8\}$.

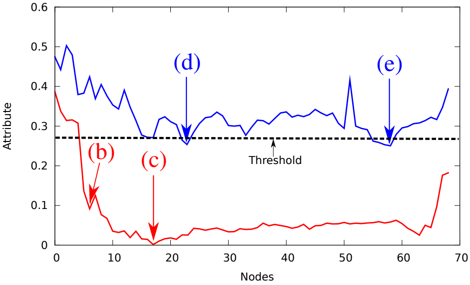
Min-tree of a tree-based image representation



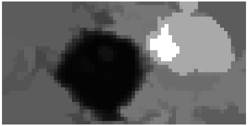
Important idea

Computing a Min-Tree on a node-weighted graph instead of a matrix image.
Easy thanks to Olena [Levillain & Géraud & Najman, ICIP, 2010], *the generic image processing platform* `http://olena.lrde.epita.fr`.

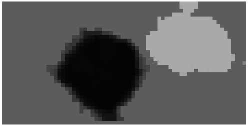
Morphological shapings



Evolution of circularity on two branches.

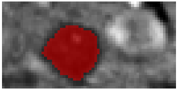


Thresholding.

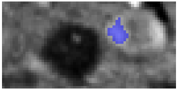


Our shaping.

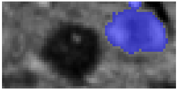
(b)



(c)

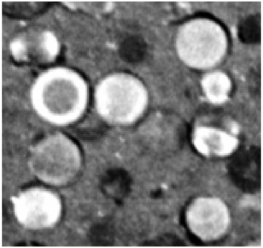


(d)

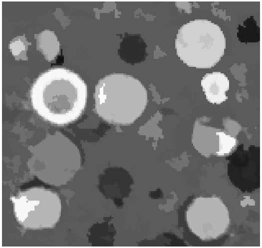


(e)

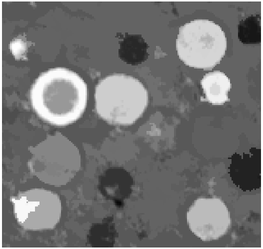
Morphological shapings



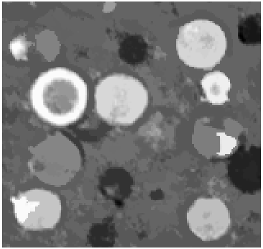
Input image.



Shaping based on $\mathcal{A}$

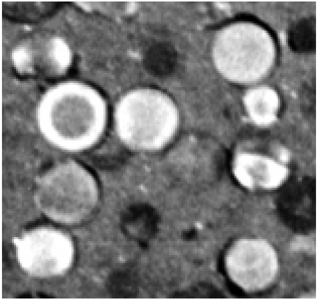


Low threshold of $\mathcal{A}$.

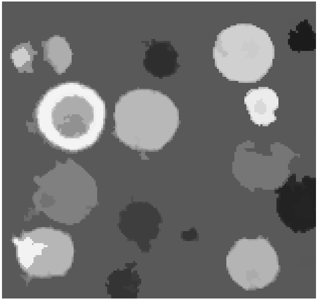


Higher threshold of $\mathcal{A}$.

Morphological shapings



Input image.



Our shaping 2.

Using a combination of attributes $\mathcal{A}$.

Encompassing classical attribute filtering strategies

Increasing attribute $\mathcal{A}$

$\mathcal{T}\mathcal{T} = \mathcal{T}$.

No need to check if the attribute is increasing or not.

Attribute thresholding for non-increasing $\mathcal{A}$

$\mathcal{A}\mathcal{A} = \mathcal{A}$,

$\mathcal{A}\mathcal{A}$ is the current level of $\mathcal{T}\mathcal{T}$.

Pruning $\mathcal{T}\mathcal{T} =$ Attribute thresholding.

Shape-based lower/upper-levelings

Shape-based lower-levelings

$\mathcal{T}$: Max-tree,
$\forall x \in E, \psi_s(f)(x) \leq f(x)$ always holds $\Rightarrow \psi_s(f)$ is a lower-leveling of f.
$\Rightarrow$ **Shape-based lower-levelings.**

Shape-based upper-levelings

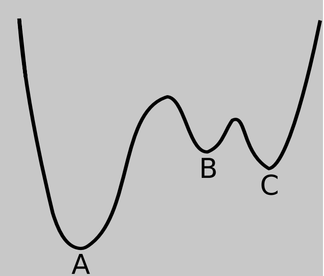
$\mathcal{T}$: Min-tree,
$\forall x \in E, \psi_s(f)(x) \geq f(x)$ always holds $\Rightarrow \psi_s(f)$ is a upper-leveling of f.
$\Rightarrow$ **Shape-based upper-levelings.**

Morphological shapings

Morphological shapings

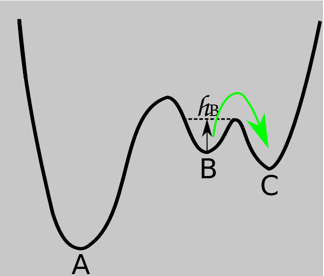
$\mathcal{T}$: Tree of shapes,
The order between $\psi_s(f)$ and f no more guaranteed, not levelings, but it is self-dual.
$\Rightarrow$ **Self-dual morphological shapings**.

Extinction-based filtering strategy



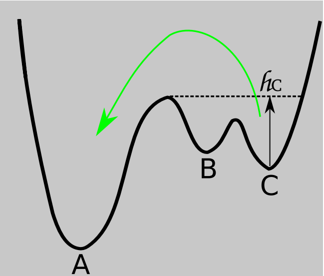
Given a strict order for the set of minima : $A \prec C \prec B$.

Extinction-based filtering strategy



B merges with C.

Extinction-based filtering strategy



C merges with A.

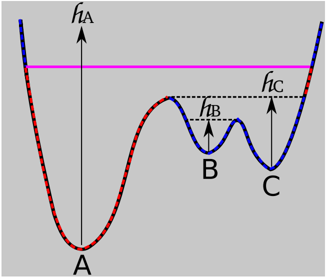
Extinction-based filtering strategy

Strategy

Preserve the **blobs of minima** whose extinction value $>$ a given value.

Advantage

Only the connected components being meaningful enough compared with their context are preserved.



Extinction value of three minima.

Application to object segmentation

Context-based estimator for object detection

[Xu & Géraud & Najman, ICIP, 2012]

$$E(u, \partial\tau) = E_{int}(u, \partial\tau) + E_{ext}(u, \partial\tau) + E_{con}(u, \partial\tau).$$

$$V(u, \mathcal{R}) \;=\; \sum_{p \in \mathcal{R}} \big(u(p) - \overline{u}(\mathcal{R})\big)^2,$$

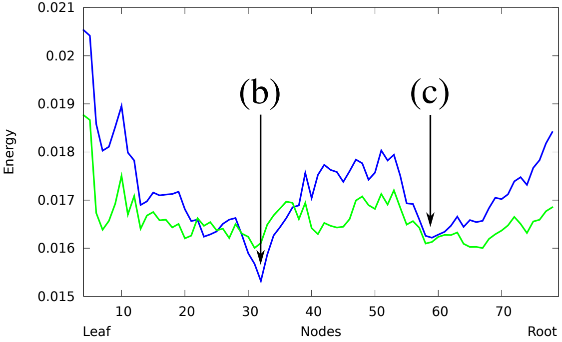
$$E_{ext}(u, \partial\tau) \;=\; \frac{V\big(u,\, \mathcal{R}^{\varepsilon}_{in}(\partial\tau)\big) \;+\; V\big(u,\, \mathcal{R}^{\varepsilon}_{out}(\partial\tau)\big)}{V\big(u,\, \mathcal{R}^{\varepsilon}_{in}(\partial\tau) \cup \mathcal{R}^{\varepsilon}_{out}(\partial\tau)\big)},$$

$$E_{int}(u, \partial\tau) \;=\; \sum_{e \in \partial\tau} |curv(u)(e)| \;/\; L(\partial\tau),$$

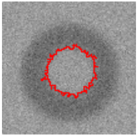
,

$$E_{con}(u, \partial\tau) \;=\; 1 \;/\; L(\partial\tau).$$

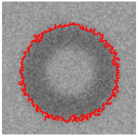
Application to object segmentation



Energy in a branch of the tree;
blue : our energy; green : snake energy.



(b)



(c)

Object detection principle

Significant minima ⇔ Objects.

Application to object segmentation

Object detection strategy

Morphological closing in the shape-space: Get rid of the spurious minima.

Any attribute $\mathcal{A}$ can be used.

Object detection results

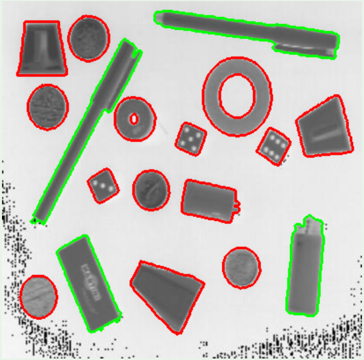
Context-based energy estimator

Input image.

Objects detected.

Object detection results

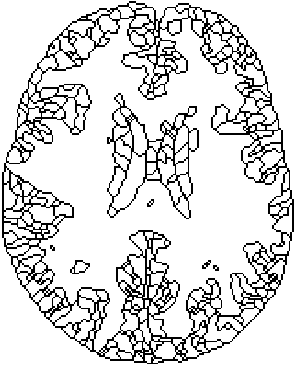
Shape attribute



Objects detected using shape attribute.
Red ones : circularity-based; Green ones : Inverse elongation-based.

Saliency map

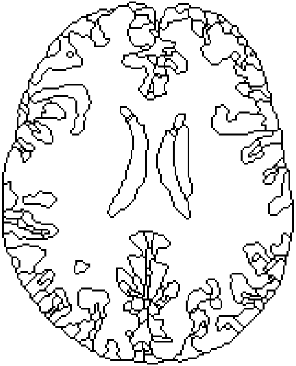
(a) Original image.



(b) Some contours.

Saliency map

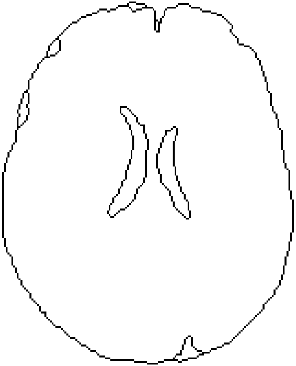
(a) Original image.



(b) Some contours.

Saliency map

(a) Original image.

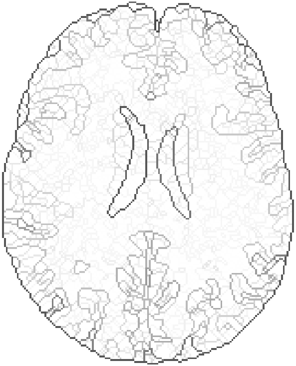


(b) Some contours.

Saliency map

Stacking the contours gives a saliency map [Najman & Schmitt, PAMI, 1996]

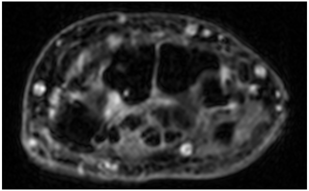
(a) Original image.



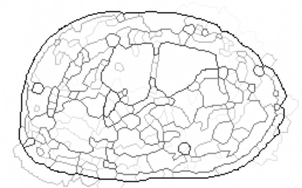
(b) A saliency map.

Different representations

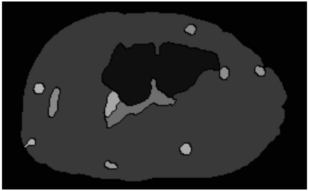
[L. Najman - JMIV - 2011] Mathematical definitions, equivalence between ultrametric watersheds, saliency maps and trees of segmentations



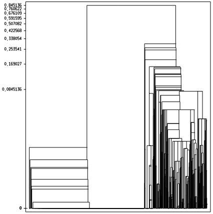
(a) Original image.



(b) Ultrametric watershed.

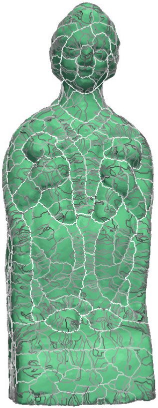


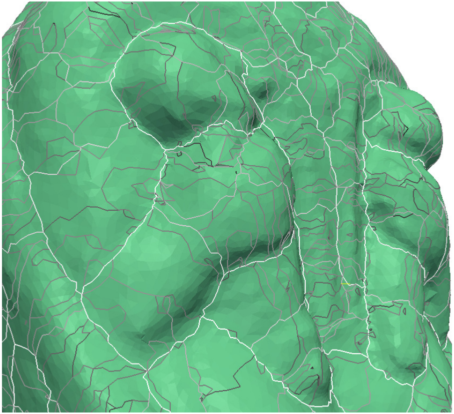
(c) One of the segmentations.



(d) Dendrogram.

Saliency maps can be computed on a mesh





Saliency maps from shape-based filterings

Idea

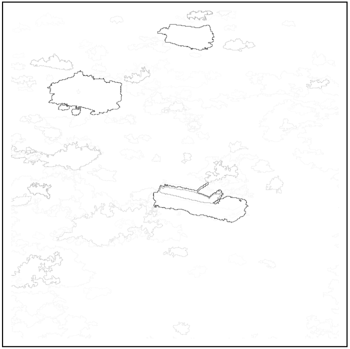
Extinction value for minima $\Leftrightarrow$ Persistence of objects $\stackrel{\mathcal{W}}{\Rightarrow}$ Saliency maps.

Strategy

$\mathcal{W}$: Weight the object contour with the maximum persistence of object that the contour belongs to.

Saliency maps from shape-based filterings

Input image.



Saliency map.

Outline

A Topological Approach to Local Feature detection

Interest point detection

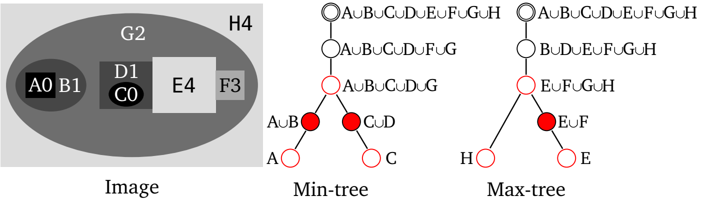
- Find a set of interesting points: DoG, Corners, ...
- Find a scale associated to each point

Find interesting regions: MSER[Matas et al., BMVC, 2002]

stability functional τ : $\tau(\mathcal{N}_k) = (|\mathcal{N}_k^+| - |\mathcal{N}_k^-|)/|\mathcal{N}_k|$.
$|.|$: cardinality; $\mathcal{N}_k^+$ and $\mathcal{N}_k^-$: resp. ancestor and descendant of node $\mathcal{N}_k$ with a prefixed range of gray level compared with $\mathcal{N}_k$.
Minima of τ are spotted as interesting regions.
Generalization: Any tree $\mathcal{T}$, any attribute $\mathcal{A}$ can be used, and the morphological closing in shape-space filters the meaningless minima.

A Topological Approach to Local Feature detection

Tree-Based Morse Regions (TBMR)



- Select critical nodes (leaves and nodes with several children)
- The scale of a critical node is the largest region containing it and topologically equivalent in its tree.

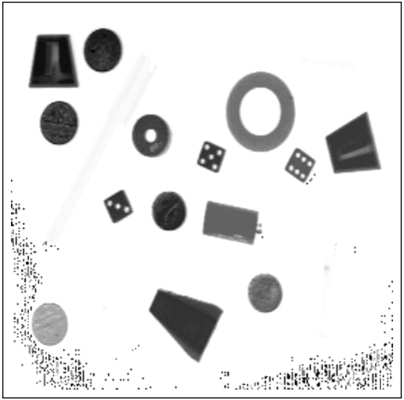
A Topological Approach to Local Feature detection

Shape-based lower/upper levelings

Input image.

Round objects based upper-leveling.

Shape-based lower/upper levelings



Difference of input image and the shape-based upper-leveling.

Blood vessels segmentation in retinal images

Important idea

1. *Use the green channel,*
2. *Black top-hat transform,*
3. *Extinction-based shape upper-leveling using circularity,*
4. *Preserved connected components are considered as blood vessels.*

Tested images

DRIVE database: Digital Retinal Images for Vessel Extraction.

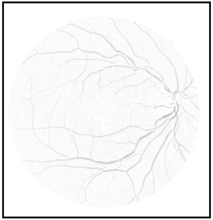
Performances measurements

1. Sensitivity and specificity : true positive and negative rate,
2. Accuracy: rate of pixels correctly classified,
3. kappa value: a statistical measure of inter-rater agreement.

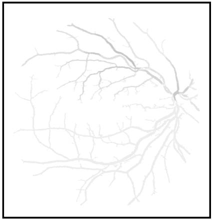
Blood vessels segmentation in retinal images

(a) Input color image.

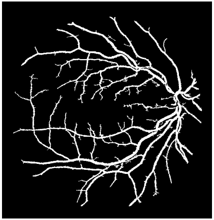
(b) Green channel.



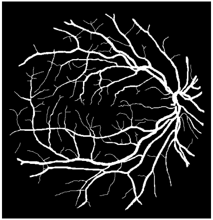
(c) Reversed black top-hat.



(d) Shape upper-leveling.



(e) Our segmentation.



(f) Manual segmentation.

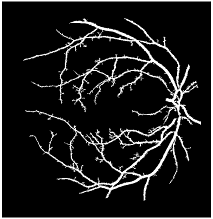
Blood vessels segmentation in retinal images

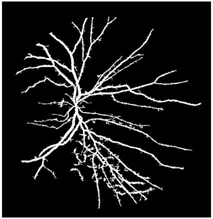

(a) Input image.

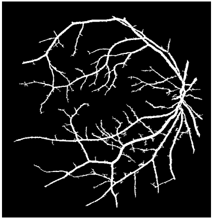

(b) Input image.

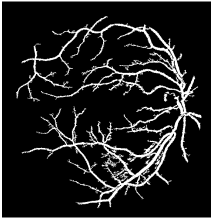

(c) Input image.


(d) Input image.


(e) Segmentation.


(f) Segmentation.


(g) Segmentation.


(h) Segmentation.

Blood vessels segmentation in retinal images

Benchmark on DRIVE database

Method	Sensitivity	Specificity	Accuracy
2nd human observer	0.7761	0.9725	0.9473 (0.0048)
mendonça	0.7344	0.9764	0.9452 (0.0062)
Our	**0.6924**	**0.9779**	**0.9413 (0.0078)**

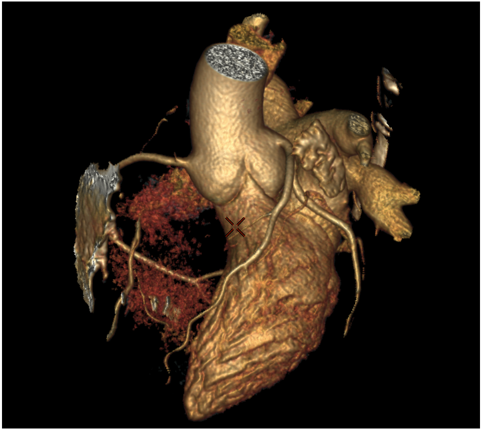
Benchmark on STARE database

Method	Sensitivity	Specificity	Accuracy
2nd human observer	0.8949	0.9390	0.9354 (0.0171)
Our	**0.7149**	**0.9749**	**0.9471 (0.0114)**
mendonça	0.6996	0.9730	0.9440 (0.0142)

Remark

This is the result of only a “simple” filtering step.

It also works in 3D: Application to coronary arteries segmentation



Path opening followed by elongation-based filtering

Optic nerve head (ONH) segmentation

Important idea

1. *Use the red channel,*
2. *Classical morphological closing by a 2D disk,*
3. *Construct the tree of shapes and calculate a specific attribute using the fuzzy theory,*
4. *The best filling ellipse of the node having the minimal attribute is identified as the ONH.*

Tested images

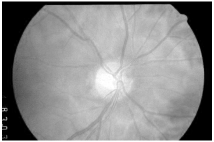
DRIONS database: Digital Retinal Images for Optic Nerve Segmentation Database.

Performances measurements

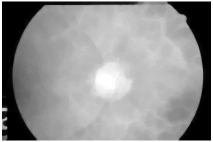
Discrepancy.

Optic nerve head (ONH) segmentation

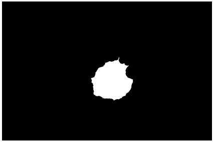
(a) Input color image.



(b) Red channel.

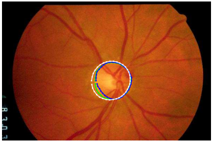


(c) Results of closing



(d) Detected CC.

(e) Segmented ONH.



(f) Manual results.

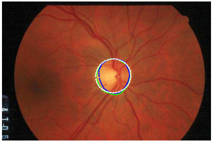
Optic nerve head (ONH) segmentation

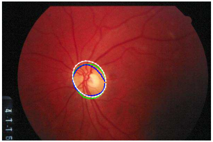

(a) Input image.

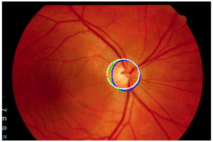

(b) Input image.

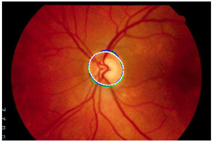

(c) Input image.


(d) Input image.


(e) ONHs.


(f) ONHs.


(g) ONHs.


(h) ONHs.

Optic nerve head (ONH) segmentation

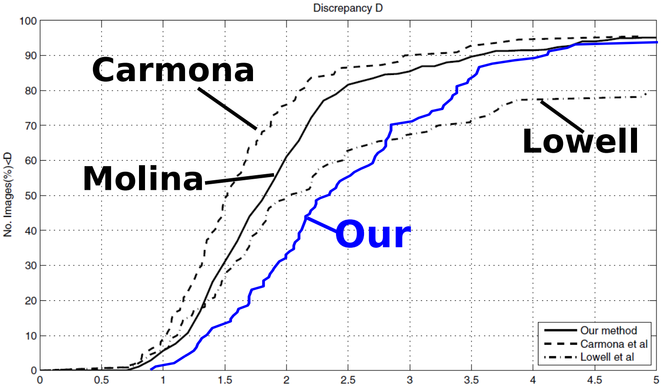
Carmona	96%
Molina	95%
Our	93.6%
Lowell	80%

Percentage of images whose discrepancy is fair

Remark

This is the result of only a "simple" filtering step.

Optic nerve head (ONH) segmentation



Accumulated discrepancy results for our detection method versus Carmona et al, Molina et al and Lowell et al.

Hierarchical simplification based on Mumford-Shah

Mumford-Shah energy with cartoon model

$$E_{\mathcal{T}} = \sum_{\partial\tau \in \mathcal{T}} \left(\sum_{p \in \mathcal{R}(\partial\tau)} \Big(u(p) - \overline{u}(\mathcal{R}(\partial\tau)) \Big)^2 + \nu L(\partial\tau) \right).$$

Attribute

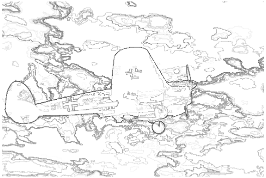
ν measures the simplification level.

Important idea

1. *Construct the tree of shapes,*
2. *Weight each node with the simplification level ν,*
3. *The saliency map yields a hierarchical simplification.*

Hierarchical simplification based on Mumford-Shah

Original.

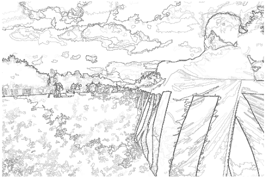


Saliency map.

Simplified.

Hierarchical simplification based on Mumford-Shah

Original.



Saliency map.

Simplified.

Hierarchical simplification based on Mumford-Shah

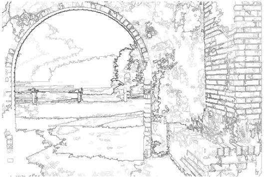
Original.

Saliency map.

Simplified.

Hierarchical simplification based on Mumford-Shah

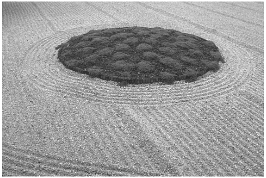
Original.



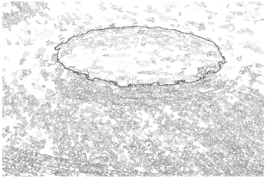
Saliency map.

Simplified.

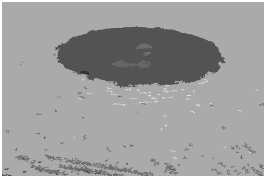
Hierarchical simplification based on Mumford-Shah



Original.



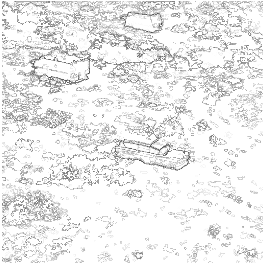
Saliency map.



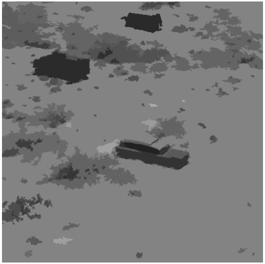
Simplified.

Hierarchical simplification based on Mumford-Shah

Original.



Saliency map.



Simplified.

Felzenswalb and Huttenlocher's algorithm

[Felzenswalb & Huttenlocher], IJCV, 2004

1. Compute a minimum spanning tree (MST) of a dissimilarity,
2. For each edge $\in$ MST linking two vertices x and y, in increasing order of their weights:
   (i) Find the region X that contains x,
   (ii) Find the region Y that contains y,
   (iii) Merge X and Y if

$$Diff(X, Y) < \min\{Int(X) + \frac{k}{|X|}, Int(Y) + \frac{k}{|Y|}\}.$$

Question

Is k a scale parameter?

Causality principle

- A contour present at a scale k_1 should be present at any scale $k_2 < k_1$.
- Not true with Felzenswalb and Huttenlocher's algorithm.

Original.

$k = 7500$ (8 regions).

$k = 9000$ (14 regions).

Application of our framework with attribute k

Answer

k is not a scale parameter.

Attribute from k

$$k = \max\left\{\left(Diff(X,Y) - Int(X)\right) \times |X|, \left(Diff(X,Y) - Int(Y)\right) \times |Y|\right\}.$$

Hierarchical image segmentation on BSDS500

Important idea

1. *Calculate the distance between neighboring pixels,*
2. *Construct a minimum spanning tree (MST),*
3. *Compute attribute k,*
4. *The saliency map yields an hierarchical image segmentation.*

Tested images

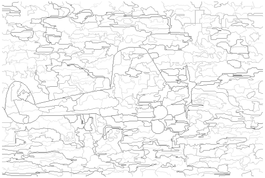
BSDS500: Berkeley Segmentation Data Set and Benchmarks 500.

Performance measurements

1. Ground-truth Covering [Arbeláez et al., PAMI, 2011],
2. Probabilistic Rand Index [Arbeláez et al., PAMI, 2011].

Hierarchical image segmentation on BSDS500

Original.



Saliency map.

Segmentation(11 regions).

Hierarchical image segmentation on BSDS500

Original.

Saliency map.

Segmentation(70 regions).

Hierarchical image segmentation on BSDS500

Original.

Saliency map.

Segmentation(20 regions).

Hierarchical image segmentation on BSDS500

Benchmarks

Our method obtains better results than the results of method of FH, and of method of Guimarães for optimal dataset scale (ODS), and for optimal image scale (OIS).

Method	GT Covering			Prob. Rand. Index	
	ODS	OIS	Best	ODS	OIS
FH	0.43	0.53	0.68	0.76	0.79
Guimarães	0.46	0.53	0.60	0.76	0.81
Ours	**0.50**	**0.57**	**0.66**	**0.77**	**0.82**

Comparison of the hierarchical segmentation obtained with Felzenswalb and Huttenlocher's algorithm, method of Guimarães et al., and our method.

Outline

Conclusion

- Object filtering

1. Encompass the state of art,
2. Shape-based lower/upper-levelings,
3. Morphological shapings.

- Object detection

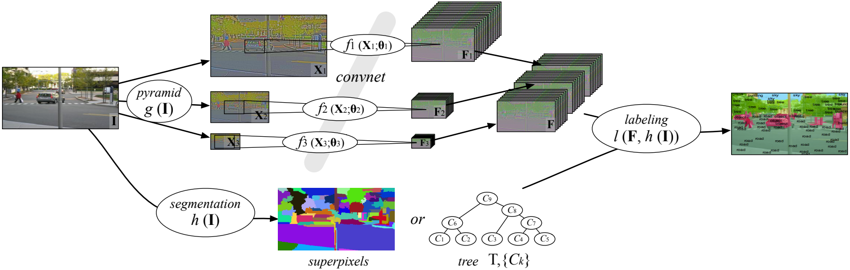
1. Context-based estimator,
2. Specific attribute $\mathcal{A}$ for ONH segmentation,
3. Saliency map.

Perspectives

- Attributes $\mathcal{A}$ and $\mathcal{AA}$,
- **Learning of the attributes**,
- Strategies of dealing with second tree $\mathcal{TT}$,
- More Properties of the morphological shapings,
- Saliency maps.

Learning Hierarchical Features for Scene Labeling

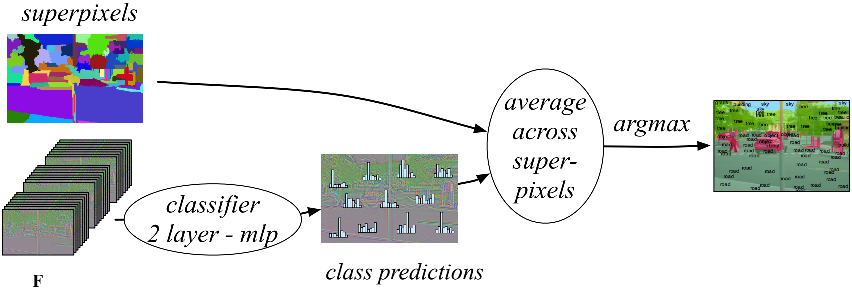
C. Farabet *et al.*, PAMI 2013



The model

Learning Hierarchical Features for Scene Labeling

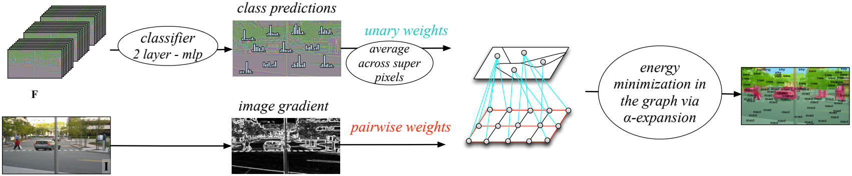
C. Farabet *et al.*, PAMI 2013



Labeling with super-pixels

Learning Hierarchical Features for Scene Labeling

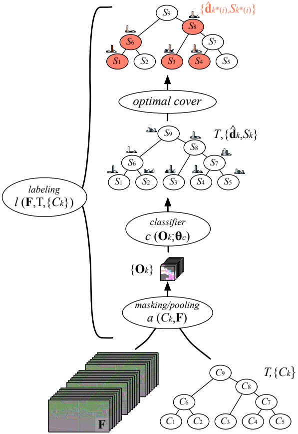
C. Farabet *et al.*, PAMI 2013



Labeling with a CRF regularization

Learning Hierarchical Features for Scene Labeling

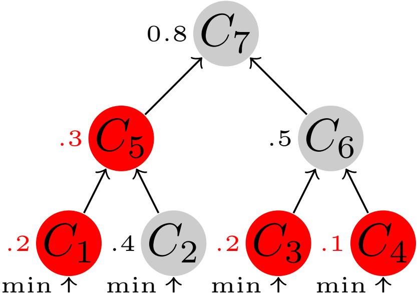
C. Farabet *et al.*, PAMI 2013



Labeling with an optimal cover tree

Learning Hierarchical Features for Scene Labeling

C. Farabet *et al.*, PAMI 2013



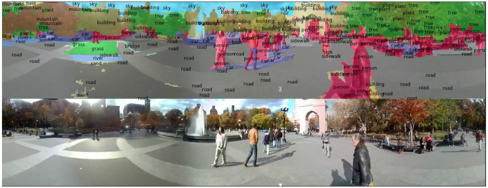
Optimal cover:

$\{C_1, C_3, C_4, C_5\}$

The optimal cover tree

Learning Hierarchical Features for Scene Labeling

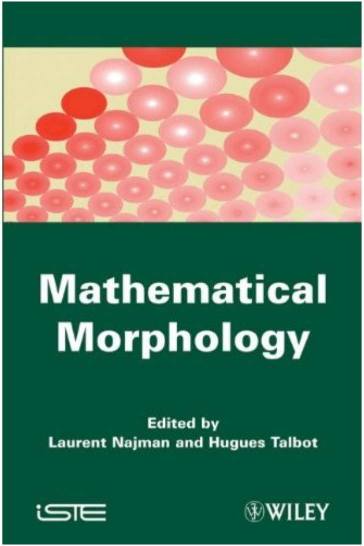
C. Farabet *et al.*, PAMI 2013



Learning Hierarchical Features for Scene Labeling

C. Farabet *et al.*, PAMI 2013

Thank for your attention !



Pink: http://pinkhq.com
Olena: http://www.lrde.epita.fr/cgi-bin/twiki/view/Olena