Saliency and hierarchies

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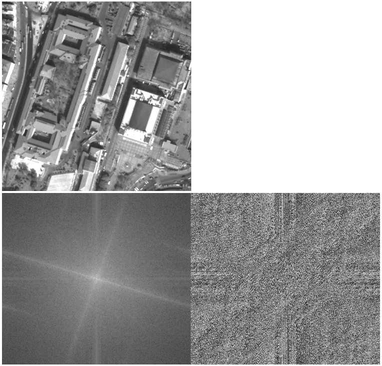
ESIEE ENGINEERING

Image representations

Decomposition into primitive or fundamental elements that can be more easily interpreted:

- Functional decomposition;
- Multiresolution decomposition;
- Multi-scale representation;
- Skeleton representation;
- Threshold decomposition;
- Hierarchical representations.



Amplitude Phase

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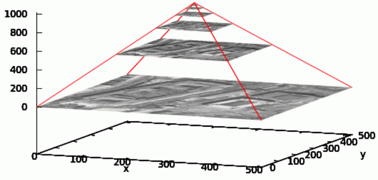

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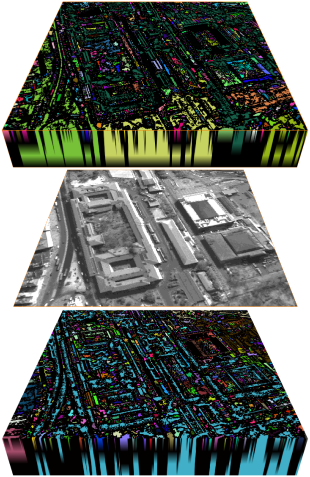

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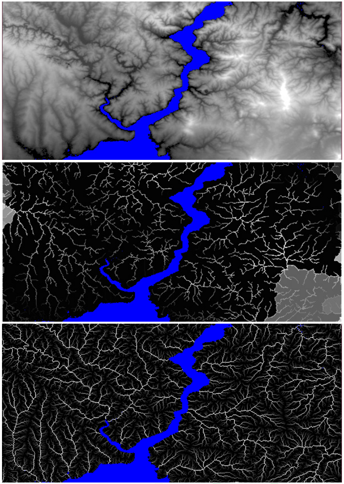

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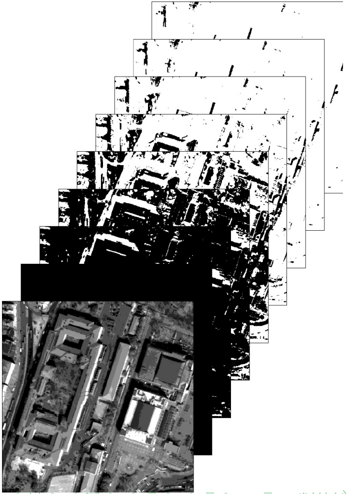

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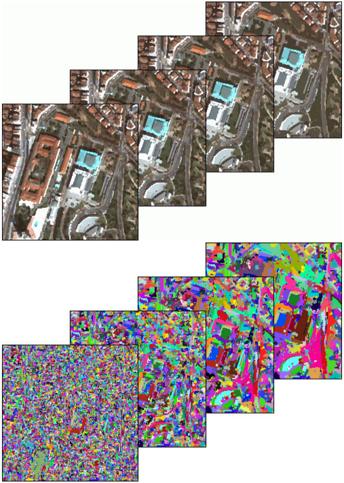

Image representations

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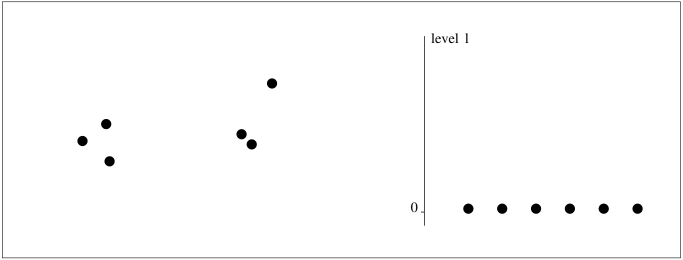
Not mutually exclusive.
Properties inherited from those of underlying operations.
Choice driven by the application needs.

Definition and representation

- Sequence of nested clusters such that a cluster at a given level is formed by unioning clusters existing at the previous level;
- The level, denoted by λ, is a non-negative real number controlling the coarseness degree of the clustering;
- Dendrograms (sometimes called taxonomic trees) are commonly used to represent hierarchies [Sokal & Sneath, 1963]:



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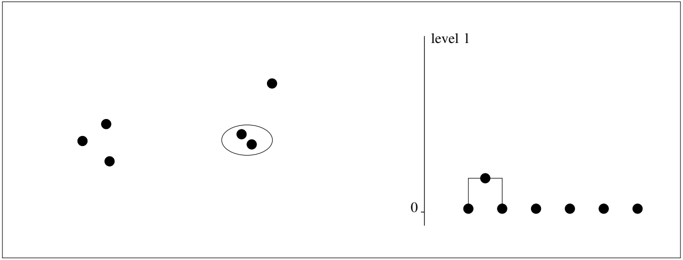

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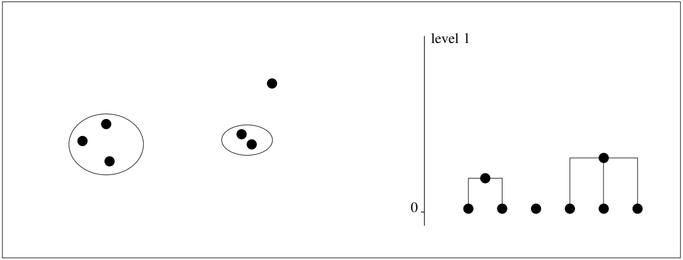

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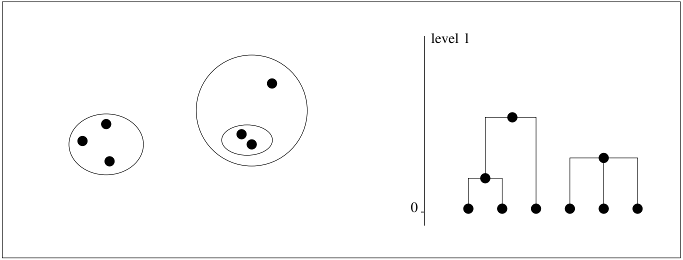

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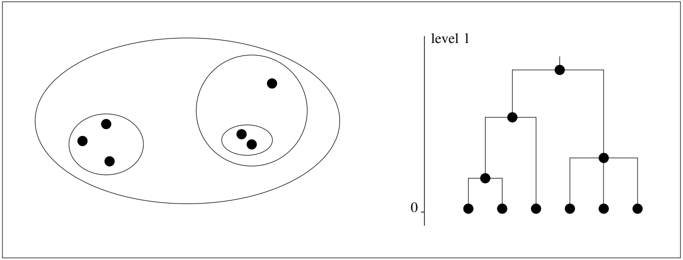
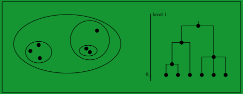


Dissimilarity



A dissimilarity measurement between the elements of a set X is a function $d^\star$ from $X \times X$ to the set of nonnegative real numbers satisfying the three following conditions:

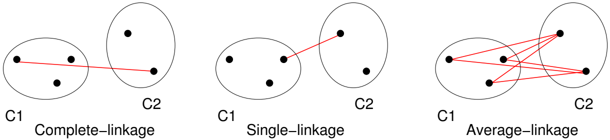
1. $d^\star(x, y) \geq 0$ for all $x, y \in X$ (i.e., positivity);
2. $d^\star(x, x) = 0$ for all $x \in X$ (i.e., nullity);
3. $d^\star(x, y) = d^\star(y, x)$ for all $x, y \in X$ (i.e., symmetry).

Let C_i and C_j denote two clusters obtained at a given level. The dissimilarity between between these two clusters is naturally defined as a function f of the dissimilarities between the objects belonging to these clusters:

$$d^\star(C_i, C_j) = f\{d^\star(x, y)) \mid x \in C_i \text{ and } y \in C_j\}.$$

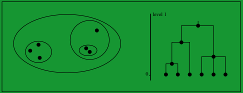
$d^\star(C_i, C_j) = f\{d^\star(x,y)) \mid x \in C_i \text{ and } y \in C_j\}$

- $f = \max$: complete-linkage clustering [Sørensen 1948];
- $f = \min$: single-linkage clustering [Sneath 1957];
- $f = \text{mean}$: average-linkage clustering [Sokal & Michener 1958].



	unicity	shape	'chaining effect'	graph setting
complete-linkage	no	compact	no	maximal cliques
single-linkage	yes	arbitrary	yes	connected comp. & **MST**
average-linkage	no	compact	no	-

Ultrametric [Jardine-Johnson, 1967]



- The distance between two objects is defined as the minimum level from which these two objects belong to the same cluster:

$$d(x, y) = \min\{\lambda \mid x \text{ and } y \text{ belong to the same cluster}\}.$$

- This distance is an ultrametric, i.e., a metric satisfying the ultrametric inequality:

$$d(x, y) \leq \max\{d(x, z), d(z, y)\}.$$

  In an ultrametric space, all triangles are either isosceles with small base or equilateral;

- There is a one-to-one correspondence between the set of hierarchical clusterings and ultrametric distances.

Some existing methods [**not** based on watersheds]

- Segmentation tree [Horowitz & Pavlidis, JACM 1976];
- Hierarchical stepwise optimisation [Beaulieu & Goldberg, PAMI 1986];
- Shortest spanning tree segmentation [Morris et al., IEE Proc. 1986];
- Pyramid of region adjacency graphs [Montanvert et al. 1991];
- Graph weighted hierarchy [Kropatsch & Haximusa, SPIE-5299 2004];
- Scale-sets: cuts minimising an energy based on complexity and distortion measures [Guigues *et al*, IJCV 2006];
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Picture Segmentation by a Tree Traversal Algorithm

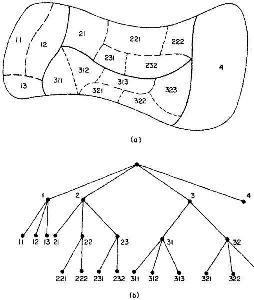

FIG. 2. (a) Example of a directed region segmentation; (b) tree representing the above segmentation

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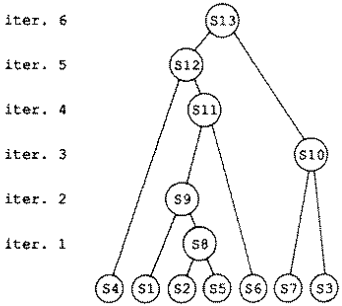

Fig. 3. Sequence of segment merges.

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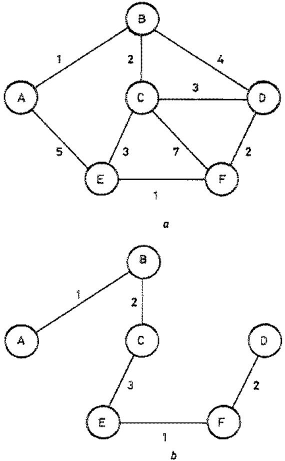

Fig. 1 *Example of a graph and its SST*
a Example graph
b SST of graph

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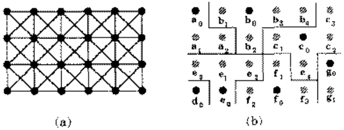

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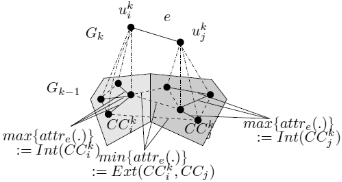

e) Internal and External contrast.

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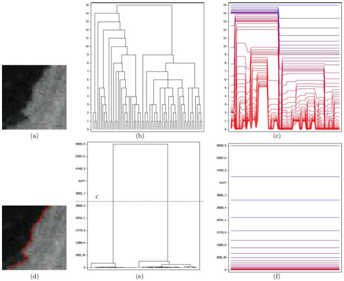

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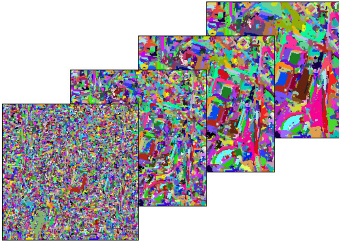
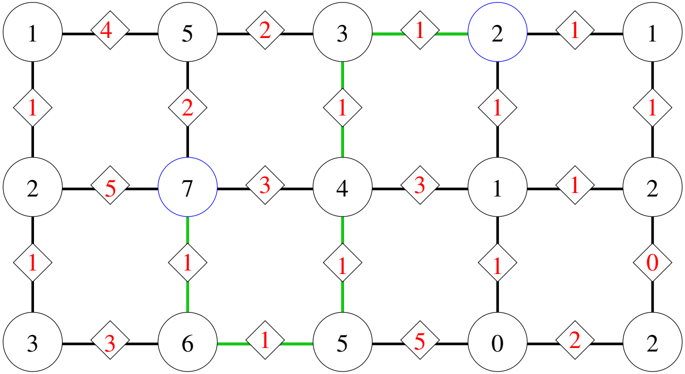


α-connectivity (introduction)

Graph $G = (V, E)$, f for the node weights, F for the edge weights.

Example with $F(\{p, q\} \in E) = | f(p) - f(q) |$:



α-connectivity

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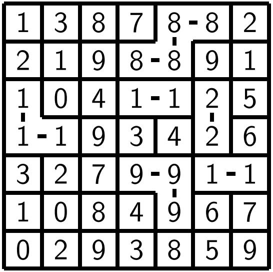
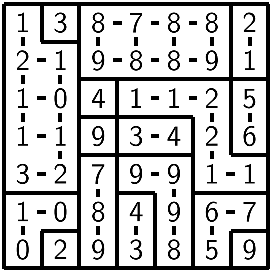
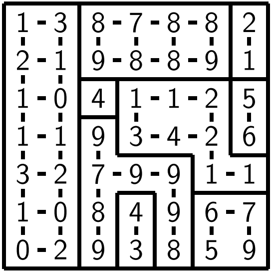


$\alpha = 0$



$\alpha = 1$



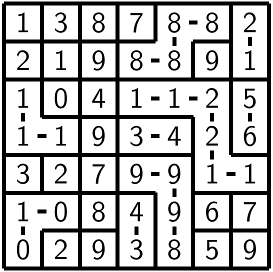
$\alpha = 2$

$\alpha\text{-CC}(p) = \{p\} \cup \Big\{ q \mid$ there exists a path $\langle p = p_1, \ldots, p_n = q \rangle$, such that $F(\{p_i, p_{i+1}\}) \leq \alpha$ for all $1 \leq i < n \Big\}$.

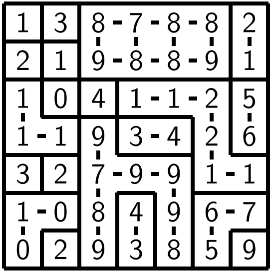
$d_A(p, q) = \min\{\alpha \mid p$ and q belong to the same $\alpha\text{-CC}\}$ is an **ultrametric**.

Constrained connectivity with global range constraint: (α, ω)-connectivity [Soille, PAMI 2008]

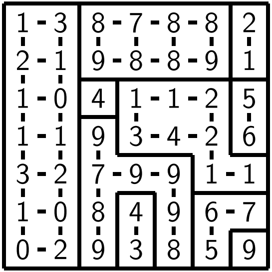
$$(\alpha, \omega)\text{-CC}(p) = \max\left\{\alpha_i\text{-CC}(p) \mid \alpha_i \leq \alpha \text{ and } \mathrm{R}\Big(\alpha_i\text{-CC}(p)\Big) \leq \omega\right\}$$



$\alpha = \omega = 1$



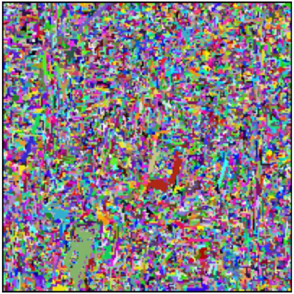
$\alpha = \omega = 2$



$\alpha = \omega = 3$

$d_\Omega(p, q) = \min\{\mathrm{R}(\alpha\text{-CC}(p)) \mid q \in \alpha\text{-CC}(p)\}$ is an ultrametric.

(α, ω)-CC



Watershed segmentation

- 1978: introduction of "the" watershed as a segmentation tool.

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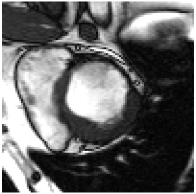

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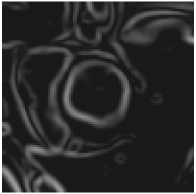

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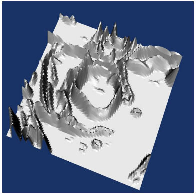

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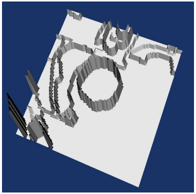

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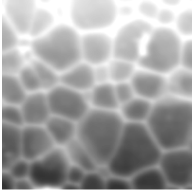
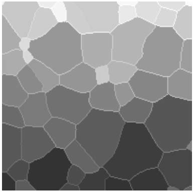
Hypothesis

- There exists numerous watershed definitions and algorithms.
- The image is seen as a graph with values on nodes.

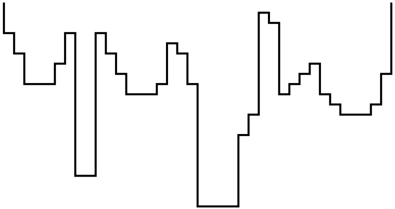
Illustration: topological watershed [Bertrand 2005, Couprie *et al.* 2005, JMIV]



Topological



Hierarchies: floodings and watersheds



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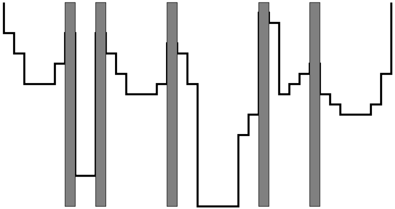

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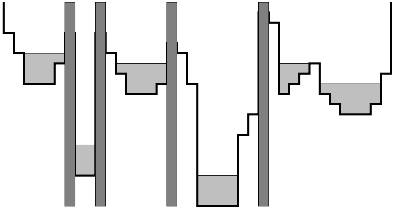

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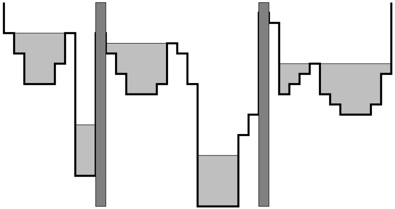

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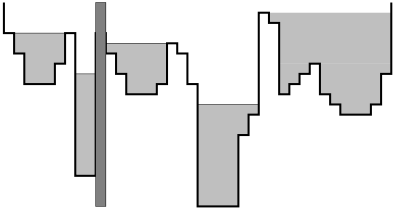

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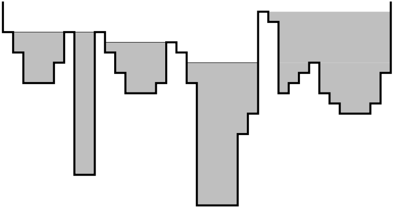

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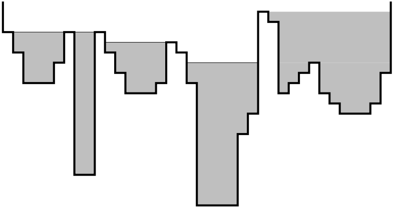
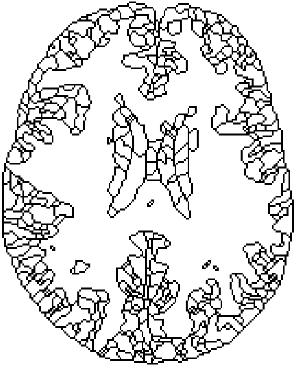


Important idea

- *There exists numerous criterions for flooding a surface.*
- *Flooding can be done through the min-(component-)tree.*
- *Among those criterions, notably: depth, surface, volume.*
- *[Beucher, ISMM, 1994 - Najman & Schmitt, PAMI, 1996 - Meyer et al., An. Telecom, 1997]*

Saliency map

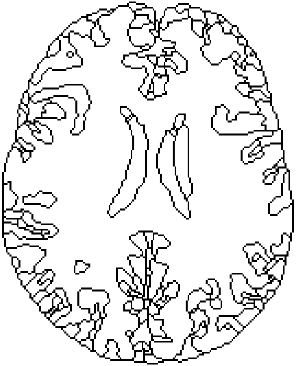
(a) Original image



(b) Some contours

Saliency map

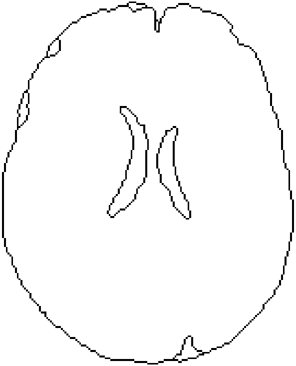
(a) Original image



(b) Some contours

Saliency map

(a) Original image

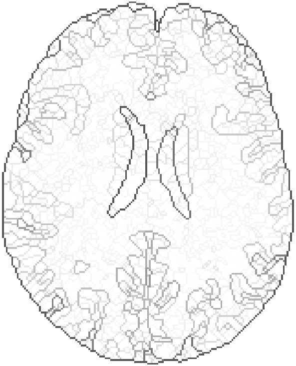


(b) Some contours

Saliency map

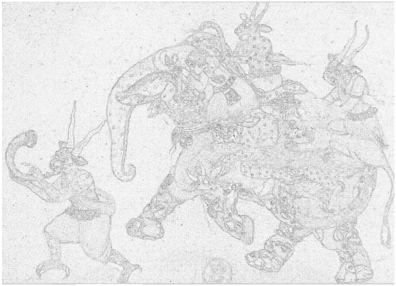
Stacking the contours gives a saliency map [Najman & Schmitt, PAMI, 1996]

(a) Original image



(b) A saliency map

Some examples



Depth driven hierarchy

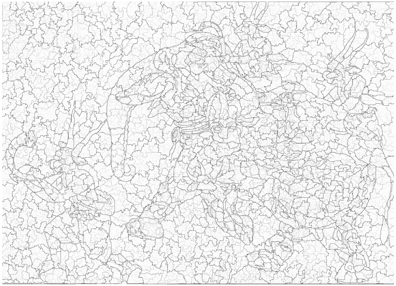
One of the segmentations

Some examples

Area driven hierarchy

One of the segmentations

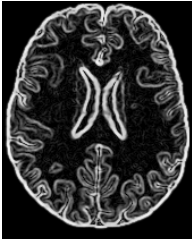
Some examples

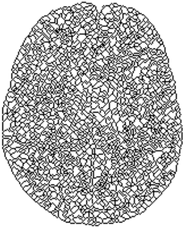


Volume driven hierarchy

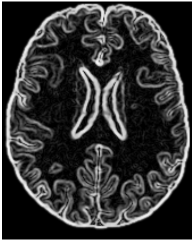
One of the segmentations

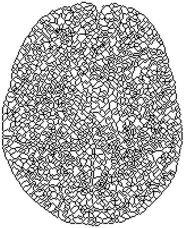
Region merging problems on pixels

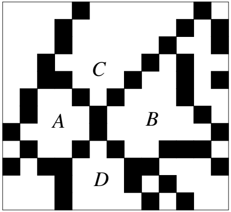




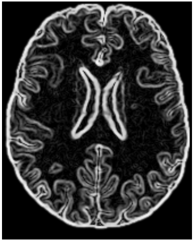
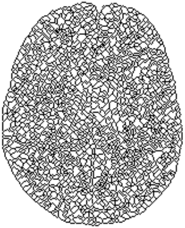
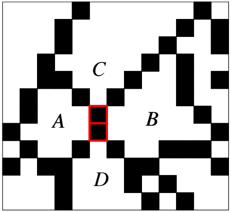
Region merging problems on pixels



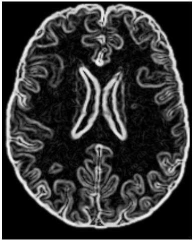


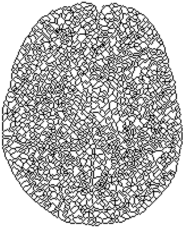


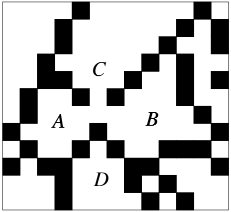
Region merging problems on pixels

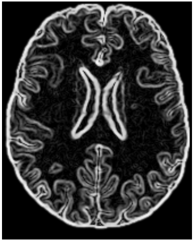
Region merging problems on pixels

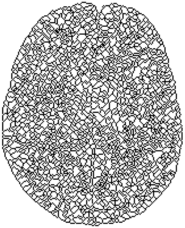


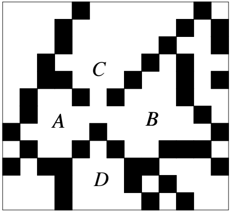




Region merging problems on pixels







Important idea

- *On the nodes:* **Fusion graphs** *[Cousty et al. - JMIV - 2008, Cousty et al. - DAM - 2008]*
- *There is no problem on edge-weighted graphs*

Main claim

Important idea

- *Any hierarchical segmentation scheme can be seen as a watershed-based hierarchical scheme.*

Main claim

Important idea

- *Any hierarchical segmentation scheme can be seen as a watershed-based hierarchical scheme.*
- *The trick is to consider edge-weighted graphs instead of node-weighted graphs.*

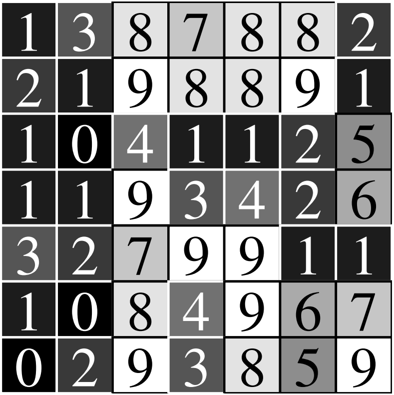
Looking at edges

1	3	8	7	8	8	2
2	1	9	8	8	9	1
1	0	4	1	1	2	5
1	1	9	3	4	2	6
3	2	7	9	9	1	1
1	0	8	4	9	6	7
0	2	9	3	8	5	9

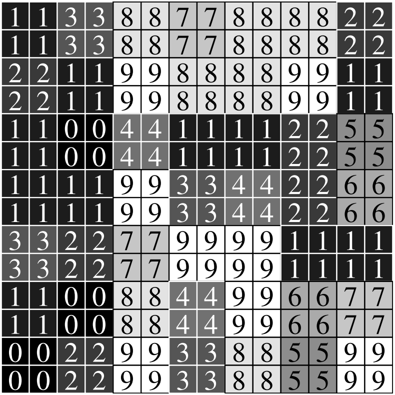
(a) Original image

Looking at edges

Doubling the graph (I like to split hairs ... and pixels)



(a) Original image



(b) Double graph

Flat zones == null gradient

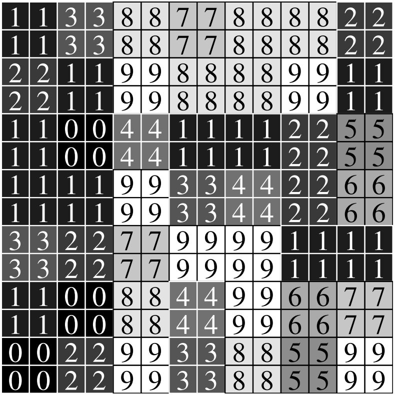
Looking at edges

1	1	3	3	8	8	7	7	8	8	8	8	2	2
1	1	3	3	8	8	7	7	8	8	8	8	2	2
2	2	1	1	9	9	8	8	8	8	9	9	1	1
2	2	1	1	9	9	8	8	8	8	9	9	1	1
1	1	0	0	4	4	1	1	1	1	2	2	5	5
1	1	0	0	4	4	1	1	1	1	2	2	5	5
1	1	1	1	9	9	3	3	4	4	2	2	6	6
1	1	1	1	9	9	3	3	4	4	2	2	6	6
3	3	2	2	7	7	9	9	9	9	1	1	1	1
3	3	2	2	7	7	9	9	9	9	1	1	1	1
1	1	0	0	8	8	4	4	9	9	6	6	7	7
1	1	0	0	8	8	4	4	9	9	6	6	7	7
0	0	2	2	9	9	3	3	8	8	5	5	9	9
0	0	2	2	9	9	3	3	8	8	5	5	9	9

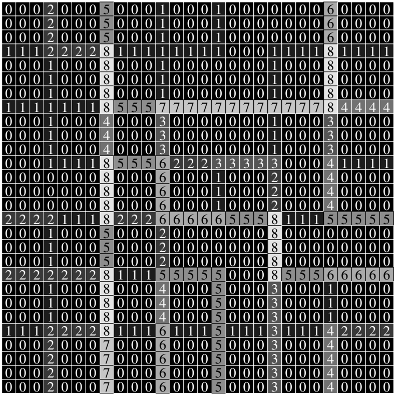
(b) Double graph

Looking at edges

Doubling the graph again (to visualize the gradient)



(b) Double graph



(c) Gradient

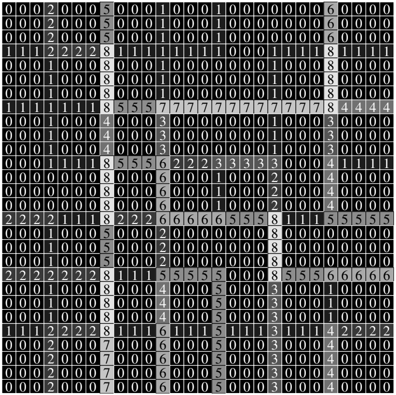
Looking at edges

0	0	0	2	0	0	0	5	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	6	0	0	0	0
0	0	0	2	0	0	0	5	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	6	0	0	0	0
0	0	0	2	0	0	0	5	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	6	0	0	0	0
1	1	1	2	2	2	2	8	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	8	1	1	1	1
0	0	0	1	0	0	0	8	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	8	0	0	0	0
0	0	0	1	0	0	0	8	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	8	0	0	0	0
0	0	0	1	0	0	0	8	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	8	0	0	0	0
1	1	1	1	1	1	1	8	5	5	5	7	7	7	7	7	7	7	7	7	7	7	7	8	4	4	4	4
0	0	0	1	0	0	0	4	0	0	0	3	0	0	0	0	0	0	0	1	0	0	0	3	0	0	0	0
0	0	0	1	0	0	0	4	0	0	0	3	0	0	0	0	0	0	0	1	0	0	0	3	0	0	0	0
0	0	0	1	0	0	0	4	0	0	0	3	0	0	0	0	0	0	0	1	0	0	0	3	0	0	0	0
0	0	0	1	1	1	1	8	5	5	5	6	2	2	2	3	3	3	3	3	0	0	0	4	1	1	1	1
0	0	0	0	0	0	0	8	0	0	0	6	0	0	0	1	0	0	0	2	0	0	0	4	0	0	0	0
0	0	0	0	0	0	0	8	0	0	0	6	0	0	0	1	0	0	0	2	0	0	0	4	0	0	0	0
0	0	0	0	0	0	0	8	0	0	0	6	0	0	0	1	0	0	0	2	0	0	0	4	0	0	0	0
2	2	2	2	1	1	1	8	2	2	2	6	6	6	6	6	5	5	5	8	1	1	1	5	5	5	5	5
0	0	0	1	0	0	0	5	0	0	0	2	0	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	5	0	0	0	2	0	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	5	0	0	0	2	0	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0
2	2	2	2	2	2	2	8	1	1	1	5	5	5	5	5	0	0	0	8	5	5	5	6	6	6	6	6
0	0	0	1	0	0	0	8	0	0	0	4	0	0	0	5	0	0	0	3	0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	8	0	0	0	4	0	0	0	5	0	0	0	3	0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	8	0	0	0	4	0	0	0	5	0	0	0	3	0	0	0	1	0	0	0	0
1	1	1	2	2	2	2	8	1	1	1	6	1	1	1	5	1	1	1	3	1	1	1	4	2	2	2	2
0	0	0	2	0	0	0	7	0	0	0	6	0	0	0	5	0	0	0	3	0	0	0	4	0	0	0	0
0	0	0	2	0	0	0	7	0	0	0	6	0	0	0	5	0	0	0	3	0	0	0	4	0	0	0	0
0	0	0	2	0	0	0	7	0	0	0	6	0	0	0	5	0	0	0	3	0	0	0	4	0	0	0	0
0	0	0	2	0	0	0	7	0	0	0	6	0	0	0	5	0	0	0	3	0	0	0	4	0	0	0	0

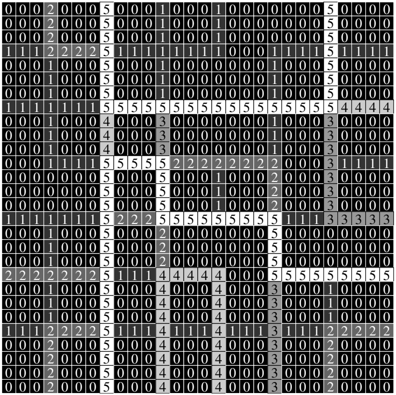
(c) Gradient

Looking at edges

Watershed: propagate the pass altitude



(c) Gradient



(d) Watershed

Looking at edges

the watershed is the saliency map of the α-connectivity

0	0	0	2	0	0	0	5	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	6	0	0	0	0
0	0	0	2	0	0	0	5	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	6	0	0	0	0
0	0	0	2	0	0	0	5	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	6	0	0	0	0
1	1	1	2	2	2	2	8	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	8	1	1	1	1
0	0	0	1	0	0	0	8	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	8	0	0	0	0
0	0	0	1	0	0	0	8	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	8	0	0	0	0
0	0	0	1	0	0	0	8	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	8	0	0	0	0
1	1	1	1	1	1	1	8	5	5	5	7	7	7	7	7	7	7	7	7	7	7	7	8	4	4	4	4
0	0	0	1	0	0	0	4	0	0	0	3	0	0	0	0	0	0	0	1	0	0	0	3	0	0	0	0
0	0	0	1	0	0	0	4	0	0	0	3	0	0	0	0	0	0	0	1	0	0	0	3	0	0	0	0
0	0	0	1	0	0	0	4	0	0	0	3	0	0	0	0	0	0	0	1	0	0	0	3	0	0	0	0
0	0	0	1	1	1	1	8	5	5	5	6	2	2	2	3	3	3	3	3	0	0	0	4	1	1	1	1
0	0	0	0	0	0	0	8	0	0	0	6	0	0	0	1	0	0	0	2	0	0	0	4	0	0	0	0
0	0	0	0	0	0	0	8	0	0	0	6	0	0	0	1	0	0	0	2	0	0	0	4	0	0	0	0
0	0	0	0	0	0	0	8	0	0	0	6	0	0	0	1	0	0	0	2	0	0	0	4	0	0	0	0
2	2	2	2	1	1	1	8	2	2	2	6	6	6	6	6	5	5	5	8	1	1	1	5	5	5	5	5
0	0	0	1	0	0	0	5	0	0	0	2	0	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	5	0	0	0	2	0	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	5	0	0	0	2	0	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0
2	2	2	2	2	2	2	8	1	1	1	5	5	5	5	5	0	0	0	8	5	5	5	6	6	6	6	6
0	0	0	1	0	0	0	8	0	0	0	4	0	0	0	5	0	0	0	3	0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	8	0	0	0	4	0	0	0	5	0	0	0	3	0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	8	0	0	0	4	0	0	0	5	0	0	0	3	0	0	0	1	0	0	0	0
1	1	1	2	2	2	2	8	1	1	1	6	1	1	1	5	1	1	1	3	1	1	1	4	2	2	2	2
0	0	0	2	0	0	0	7	0	0	0	6	0	0	0	5	0	0	0	3	0	0	0	4	0	0	0	0
0	0	0	2	0	0	0	7	0	0	0	6	0	0	0	5	0	0	0	3	0	0	0	4	0	0	0	0
0	0	0	2	0	0	0	7	0	0	0	6	0	0	0	5	0	0	0	3	0	0	0	4	0	0	0	0
0	0	0	2	0	0	0	7	0	0	0	6	0	0	0	5	0	0	0	3	0	0	0	4	0	0	0	0

(c) Gradient

0	0	0	2	0	0	0	5	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	5	0	0	0	0
0	0	0	2	0	0	0	5	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	5	0	0	0	0
0	0	0	2	0	0	0	5	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	5	0	0	0	0
1	1	1	2	2	2	2	5	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	5	1	1	1	1
0	0	0	1	0	0	0	5	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	5	0	0	0	0
0	0	0	1	0	0	0	5	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	5	0	0	0	0
0	0	0	1	0	0	0	5	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	5	0	0	0	0
1	1	1	1	1	1	1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	4	4	4	4
0	0	0	1	0	0	0	4	0	0	0	3	0	0	0	0	0	0	0	1	0	0	0	3	0	0	0	0
0	0	0	1	0	0	0	4	0	0	0	3	0	0	0	0	0	0	0	1	0	0	0	3	0	0	0	0
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0	0	0	0	0	0	0	5	0	0	0	5	0	0	0	1	0	0	0	2	0	0	0	3	0	0	0	0
1	1	1	1	1	1	1	5	2	2	2	5	5	5	5	5	5	5	5	5	1	1	1	3	3	3	3	3
0	0	0	1	0	0	0	5	0	0	0	2	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	5	0	0	0	2	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	5	0	0	0	2	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0	0
2	2	2	2	2	2	2	5	1	1	1	4	4	4	4	4	0	0	0	5	5	5	5	5	5	5	5	5
0	0	0	1	0	0	0	5	0	0	0	4	0	0	0	4	0	0	0	3	0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	5	0	0	0	4	0	0	0	4	0	0	0	3	0	0	0	1	0	0	0	0
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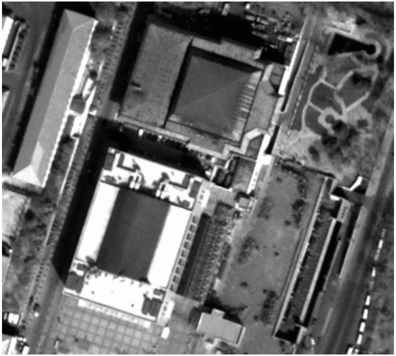
1	1	1	2	2	2	2	5	1	1	1	4	1	1	1	4	1	1	1	3	1	1	1	2	2	2	2	2
0	0	0	2	0	0	0	5	0	0	0	4	0	0	0	4	0	0	0	3	0	0	0	2	0	0	0	0
0	0	0	2	0	0	0	5	0	0	0	4	0	0	0	4	0	0	0	3	0	0	0	2	0	0	0	0
0	0	0	2	0	0	0	5	0	0	0	4	0	0	0	4	0	0	0	3	0	0	0	2	0	0	0	0
0	0	0	2	0	0	0	5	0	0	0	4	0	0	0	4	0	0	0	3	0	0	0	2	0	0	0	0

(d) Watershed

Application



Original image

α-connectivity saliency map

Main result - a new class of watersheds: ultrametric watersheds

Theorem

- *Saliency maps can be characterized as ultrametric watersheds*
- *Ultrametric watersheds have a computable definition*
- *There exists a bijection between the set of ultrametric watersheds and the set of hierarchical segmentations.*
- *[Najman, ISMM 2009]*

Ultrametric watersheds: formal definitions

- If $S \subset E$, $\overline{S} = E \setminus S$.
- $F[\lambda] = \{v \in E \mid F(v) \leq \lambda\}$.
- An edge $u \in \overline{E(X)}$ is said to be *W-simple (for X)* if X has the same number of connected components as $X + u$.
- An edge u such that $F(u) = \lambda$ is said to be *W-destructible (for F) with lowest value λ_0* if there exists λ_0 such that, for all λ_1, $\lambda_0 < \lambda_1 \leq \lambda$, u is W-simple for $F[\lambda_1]$ and if u is not W-simple for $F[\lambda_0]$.
- A *topological watershed (on G)* is a map that contains no W-destructible edges.
- A map F is an *ultrametric watershed* if F is a topological watershed, and if furthemore, for any minimum X of F, $F(X) = 0$.

Ultrametric watersheds: some properties

The *connection value* is the number $F(x,y) = \min_{\pi \in \Pi(x,y)} \max\{F(u) | u \in \pi\}$, where $\Pi(x,y)$ is the set of all paths linking x to y in G. If X and Y are two subgraphs of G, we set $F(X,Y) = \min\{F(x,y) \mid x \in X, y \in Y\}$.

Theorem

A map F is a topological watershed if and only if:

(i) *Its minima form a segmentation of G;*

(ii) *for any edge $v = \{x,y\}$, if there exist X and Y in $\mathcal{M}(F)$, $X \neq Y$, such that $x \in V(X)$ and $y \in V(Y)$, then $F(v) = F(X,Y)$.*

Property

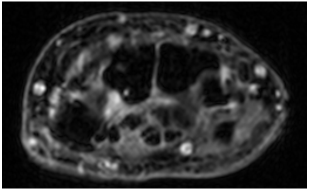
A map F is an ultrametric watershed if and only if for all $\lambda \geq 0$, $F[\lambda]$ is a segmentation of G.

Illustration of main theorem

Novel potential methodology

Illustration of main theorem

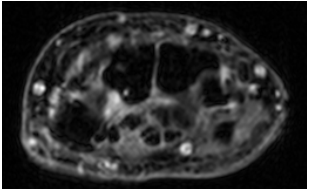
Novel potential methodology



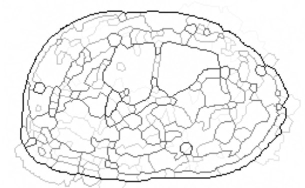
(a) Original image

Illustration of main theorem

Novel potential methodology



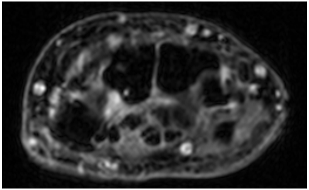
(a) Original image



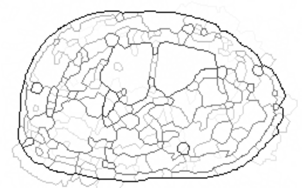
(b) Ultrametric watershed

Illustration of main theorem

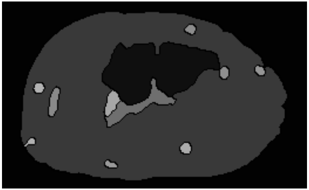
Novel potential methodology



(a) Original image



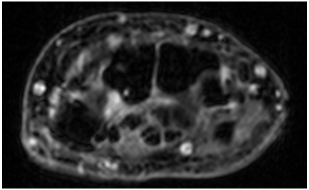
(b) Ultrametric watershed



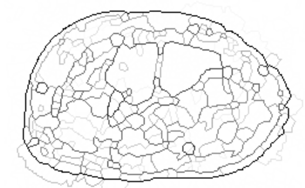
(c) One of the segmentations

Illustration of main theorem

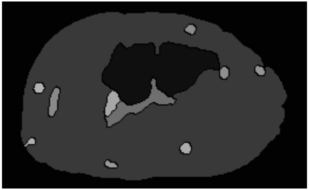
Novel potential methodology



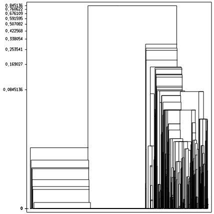
(a) Original image



(b) Ultrametric watershed



(c) One of the segmentations



(d) Dendrogram

Illustration of main theorem

Result

Main theorem: the dendrogram can be replaced by an ultrametric watershed

Looking at min-tree of edge-maps

The min-tree of a saliency map : the connected components of all the thresholds of a saliency map

0	0	0	2	0	0	0	5	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	5	0	0	0	0
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0	0	0	2	0	0	0	5	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	5	0	0	0	0
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0	0	0	1	0	0	0	5	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	5	0	0	0	0
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0	0	0	0	0	0	0	5	0	0	0	5	0	0	0	1	0	0	0	2	0	0	0	3	0	0	0	0
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0	0	0	1	0	0	0	5	0	0	0	2	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0	0
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0	0	0	1	0	0	0	5	0	0	0	4	0	0	0	4	0	0	0	3	0	0	0	1	0	0	0	0
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0	0	0	2	0	0	0	5	0	0	0	4	0	0	0	4	0	0	0	3	0	0	0	2	0	0	0	0
0	0	0	2	0	0	0	5	0	0	0	4	0	0	0	4	0	0	0	3	0	0	0	2	0	0	0	0
0	0	0	2	0	0	0	5	0	0	0	4	0	0	0	4	0	0	0	3	0	0	0	2	0	0	0	0
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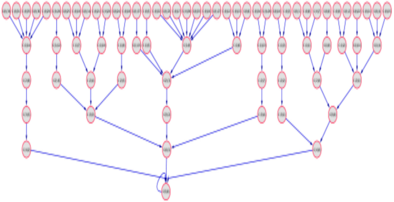

Looking at min-tree of edge-maps

It is a dendrogram (with more information)

0	0	0	2	0	0	0	5	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	5	0	0	0	0
0	0	0	2	0	0	0	5	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	5	0	0	0	0
0	0	0	2	0	0	0	5	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	5	0	0	0	0
1	1	1	2	2	2	2	5	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	5	1	1	1	1
0	0	0	1	0	0	0	5	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	5	0	0	0	0
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0	0	0	1	0	0	0	5	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	5	0	0	0	0
1	1	1	1	1	1	1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	4	4	4	4
0	0	0	1	0	0	0	4	0	0	0	3	0	0	0	0	0	0	0	1	0	0	0	3	0	0	0	0
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0	0	0	1	0	0	0	4	0	0	0	3	0	0	0	0	0	0	0	1	0	0	0	3	0	0	0	0
0	0	0	1	1	1	1	5	5	5	5	5	2	2	2	2	2	2	2	2	0	0	0	3	1	1	1	1
0	0	0	0	0	0	0	5	0	0	0	5	0	0	0	1	0	0	0	2	0	0	0	3	0	0	0	0
0	0	0	0	0	0	0	5	0	0	0	5	0	0	0	1	0	0	0	2	0	0	0	3	0	0	0	0
0	0	0	0	0	0	0	5	0	0	0	5	0	0	0	1	0	0	0	2	0	0	0	3	0	0	0	0
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0	0	0	1	0	0	0	5	0	0	0	2	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	5	0	0	0	2	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	5	0	0	0	2	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0	0
2	2	2	2	2	2	2	5	1	1	1	4	4	4	4	4	0	0	0	5	5	5	5	5	5	5	5	5
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0	0	0	1	0	0	0	5	0	0	0	4	0	0	0	4	0	0	0	3	0	0	0	1	0	0	0	0
1	1	1	2	2	2	2	5	1	1	1	4	1	1	1	4	1	1	1	3	1	1	1	2	2	2	2	2
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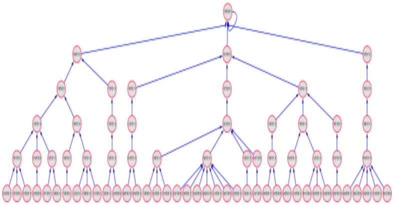

Constrained connectivity as a flooding

- The range constraint is increasing on the min-tree of the gradient

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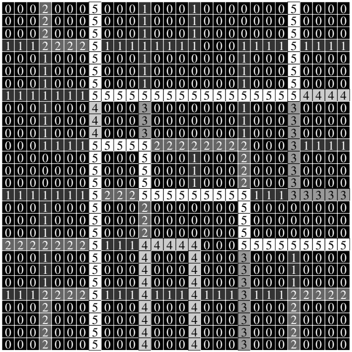

α-connectivity

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0	0	0	2	0	0	0	5	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	5	0	0	0	0
0	0	0	2	0	0	0	5	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	5	0	0	0	0
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0	0	0	2	0	0	0	5	0	0	0	4	0	0	0	4	0	0	0	3	0	0	0	2	0	0	0	0
0	0	0	2	0	0	0	5	0	0	0	4	0	0	0	4	0	0	0	3	0	0	0	2	0	0	0	0

α-connectivity

0	0	0	3	0	0	0	9	0	0	0	2	0	0	0	2	0	0	0	0	0	0	0	9	0	0	0	0
0	0	0	3	0	0	0	9	0	0	0	2	0	0	0	2	0	0	0	0	0	0	0	9	0	0	0	0
0	0	0	3	0	0	0	9	0	0	0	2	0	0	0	2	0	0	0	0	0	0	0	9	0	0	0	0
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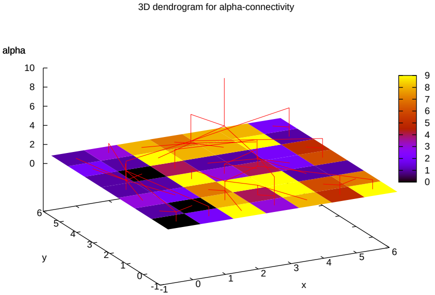
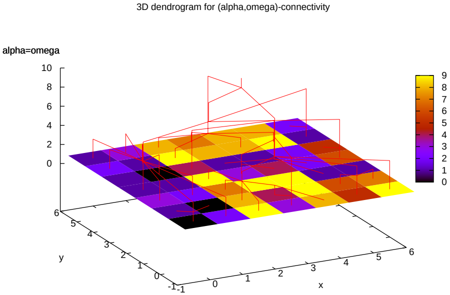
3	3	3	3	3	3	3	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	6	6	6	6
0	0	0	3	0	0	0	6	0	0	0	5	0	0	0	0	0	0	0	1	0	0	0	5	0	0	0	0
0	0	0	3	0	0	0	6	0	0	0	5	0	0	0	0	0	0	0	1	0	0	0	5	0	0	0	0
0	0	0	3	0	0	0	6	0	0	0	5	0	0	0	0	0	0	0	1	0	0	0	5	0	0	0	0
0	0	0	3	3	3	3	9	9	9	9	9	3	3	3	3	3	3	3	3	0	0	0	5	1	1	1	1
0	0	0	0	0	0	0	9	0	0	0	9	0	0	0	1	0	0	0	3	0	0	0	5	0	0	0	0
0	0	0	0	0	0	0	9	0	0	0	9	0	0	0	1	0	0	0	3	0	0	0	5	0	0	0	0
0	0	0	0	0	0	0	9	0	0	0	9	0	0	0	1	0	0	0	3	0	0	0	5	0	0	0	0
3	3	3	3	3	3	3	9	2	2	2	9	9	9	9	9	9	9	9	9	1	1	1	5	5	5	5	5
0	0	0	3	0	0	0	9	0	0	0	2	0	0	0	0	0	0	0	9	0	0	0	0	0	0	0	0
0	0	0	3	0	0	0	9	0	0	0	2	0	0	0	0	0	0	0	9	0	0	0	0	0	0	0	0
0	0	0	3	0	0	0	9	0	0	0	2	0	0	0	0	0	0	0	9	0	0	0	0	0	0	0	0
3	3	3	3	3	3	3	9	2	2	2	6	6	6	6	6	0	0	0	9	9	9	9	9	9	9	9	9
0	0	0	1	0	0	0	9	0	0	0	6	0	0	0	6	0	0	0	4	0	0	0	2	0	0	0	0
0	0	0	1	0	0	0	9	0	0	0	6	0	0	0	6	0	0	0	4	0	0	0	2	0	0	0	0
0	0	0	1	0	0	0	9	0	0	0	6	0	0	0	6	0	0	0	4	0	0	0	2	0	0	0	0
1	1	1	3	3	3	3	9	2	2	2	6	1	1	1	6	1	1	1	4	2	2	2	4	4	4	4	4
0	0	0	3	0	0	0	9	0	0	0	6	0	0	0	6	0	0	0	4	0	0	0	4	0	0	0	0
0	0	0	3	0	0	0	9	0	0	0	6	0	0	0	6	0	0	0	4	0	0	0	4	0	0	0	0
0	0	0	3	0	0	0	9	0	0	0	6	0	0	0	6	0	0	0	4	0	0	0	4	0	0	0	0
0	0	0	3	0	0	0	9	0	0	0	6	0	0	0	6	0	0	0	4	0	0	0	4	0	0	0	0

Constrained connectivity

Spatially rooted dendrogram (3D dendrograms)





Application

α-connectivity

Constrained connectivity

Different (hierarchical) image representations

Important idea

Choose what is best adapted to the problem at hand

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- **Ultrametric watersheds / saliency maps** == to **see** any hierarchy as an **image**

The problem of transition pixels

Difficulty

Transition pixels are present in any hierarchical scheme

0	1	0	7	8	7	8
1	0	1	6	7	8	7
0	1	0	5	8	7	8
1	0	1	4	7	8	7
0	1	0	3	8	7	8
1	0	1	2	7	8	7
0	1	0	1	8	7	8

An image

0	1	0	7	8	7	8
1	0	1	6	7	8	7
0	1	0	5	8	7	8
1	0	1	4	7	8	7
0	1	0	3	8	7	8
1	0	1	2	7	8	7
0	1	0	1	8	7	8

Constrained connectivity saliency map

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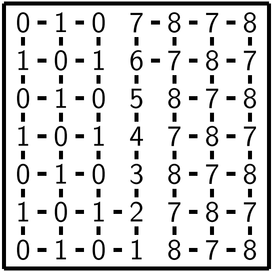
1	0	1	2	7	8	7
0	1	0	1	8	7	8

An image



The ramp
(of gradient == 1)

Application

Important idea

Seing a hierarchy as an image allows to apply any operator to solve a difficulty

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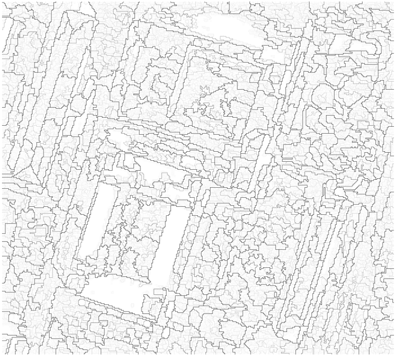
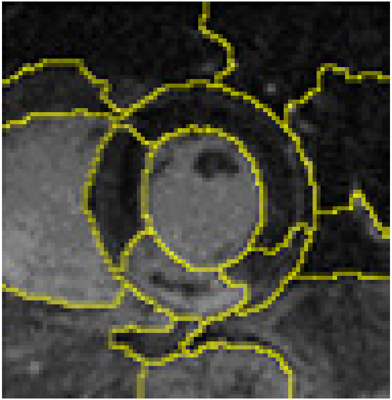
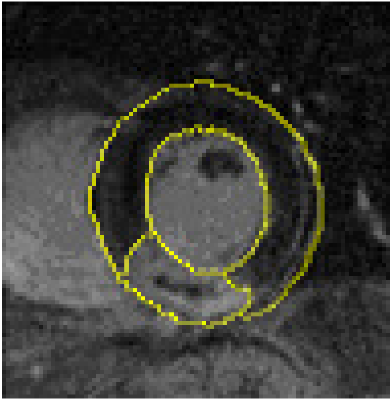


area-filtering

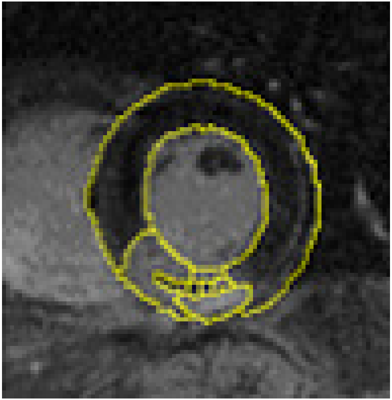
Example: local resegmentation



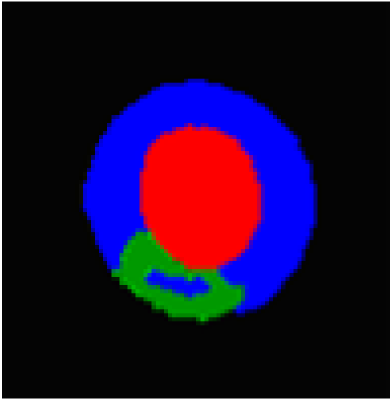
Example: local resegmentation



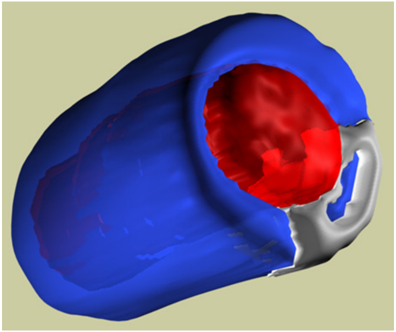
Example: local resegmentation



Example: local resegmentation



Example: local resegmentation



Example: magic-wand driven hierarchy

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Example: magic-wand driven hierarchy

Example: hierarchical lasso

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Example: hierarchical brush

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Conclusion

- Limitations of hierarchical schemes: need for overlapping clusters?
- Further benefit from huge litterature on data clustering expected.